Significant Figures

It is important to be honest when reporting a measurement, so that it does not appear to be more accurate than the equipment used to make the measurement allows. We can achieve this by controlling the number of digits, or **significant figures**, used to report the measurement.

Determining the Number of Significant Figures

The number of significant figures in a measurement, such as 2.531, is equal to the number of digits that are known with some degree of confidence (2, 5, and 3) plus the last digit (1), which is an estimate or approximation. As we improve the sensitivity of the equipment used to make a measurement, the number of significant figures increases.

Postage scale	3 ± 1 g	1 significant figure
Two-pan balance	2.53 ± 0.01 g	3 significant figure
Analytical balance	2.531 ± 0.001 g	4 significant figure

Rules for counting significant figures are summarized below.

Zeros *within* a number are always significant. Both 4308 and 40.05 contain four significant figures.

Zeros that do nothing but set the decimal point are not significant. Thus, 470,000 has two significant figures.

Trailing zeros that aren't needed to hold the decimal point are significant. For example, 4.00 has three significant figures.

If you are not sure whether a digit is significant, assume that it isn't. For example, if the directions for an experiment read: "Add the sample to 400 mL of water," assume the volume of water is known to one significant figure.

Addition and Subtraction with Significant Figures

When combining measurements with different degrees of accuracy and precision, *the accuracy of the final answer can be no greater than the least accurate measurement.* This principle can be translated into a simple rule for addition and subtraction: **When measurements are added or subtracted, the answer can contain no more decimal places than the least accurate measurement.**

Example:

$$\begin{array}{ll} 150.0 & \text{g } H_2O \\ \underline{+\ 0.507} & \underline{\text{g salt}} \\ 150.5 & \text{g solution} \end{array}$$

Multiplication and Division with Significant Figures

The same principle governs the use of significant figures in multiplication and division: the final result can be no more accurate than the least accurate measurement. In this case, however, we count the significant figures in each measurement, not the number of decimal places: **When measurements are multiplied or divided, the answer can contain no more significant figures than the least accurate measurement.**

Example:

Let's calculate the cost of copper in an old penny that is pure copper. Let's assume that the penny has a mass of 2.531 grams, that it is essentially pure copper, and that the price of copper is 67 cents per pound. We can start by converting from grams to pounds.

$$2.531\ g \times \frac{1\ lb}{453.6\ g} = 0.005580\ lb$$

We then use the price of a pound of copper to calculate the cost of the copper metal.

$$0.005580\ lb \times \frac{67\ ¢}{1\ lb} = 0.3749\ ¢$$

There are four significant figures in both the mass of the penny (2.531) and the number of grams in a pound (453.6). However, there are only two significant figures in the price of copper, so the final answer can only have two significant figures.

Rounding Off

When the answer to a calculation contains too many significant figures, it must be rounded off.

There are 10 digits that can occur in the last decimal place in a calculation. One way of rounding off involves *underestimating* the answer for five of these digits (0, 1, 2, 3, and 4) and *overestimating* the answer for the other five (5, 6, 7, 8, and 9). This approach to rounding off is summarized as follows.

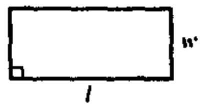
If the digit is smaller than 5, drop this digit and leave the remaining number unchanged. Thus, 1.684 becomes 1.68.

If the digit is 5 or larger, drop this digit and add 1 to the preceding digit. Thus, 1.247 becomes 1.25.

GEOMETRIC FORMULAS

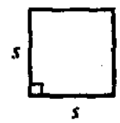
Rectangle

Perimeter: $P = 2 \cdot l + 2 \cdot w$
Area: $A = l \cdot w$



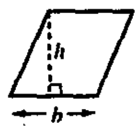
Square

Perimeter: $P = 4 \cdot s$
Area: $A = s^2$



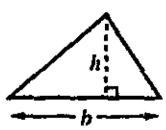
Parallelogram

Perimeter: Add the lengths of the sides.
Area: $A = b \cdot h$



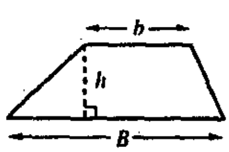
Triangle

Perimeter: Add the lengths of the sides.
Area: $A = \frac{1}{2} \cdot b \cdot h$

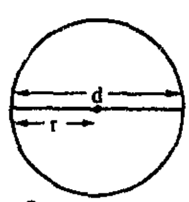


Trapezoid

Perimeter: Add the lengths of the sides.
Area: $A = \frac{1}{2} \cdot h \cdot (b + B)$



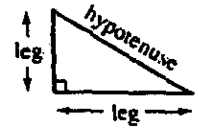
Circle



Diameter: $d = 2 \cdot r$
Radius: $r = \frac{d}{2}$
Circumference: $C = \pi \cdot d$ or $C = 2 \cdot \pi \cdot r$
Area: $A = \pi \cdot r^2$
Use 3.14 as the approximate value of π.

Right Triangle

Hypotenuse $= \sqrt{(\text{leg})^2 + (\text{leg})^2}$
Leg $= \sqrt{(\text{hypotenuse})^2 - (\text{leg})^2}$

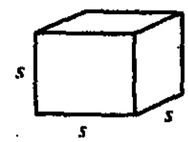


Rectangular Solid

Volume: $V = l \cdot w \cdot h$
Surface Area: $A = 2HW + 2LW + 2LH$

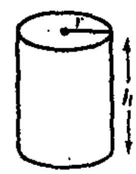
Cube

Volume: $V = s^3$
Surface Area: $A = 6s^2$



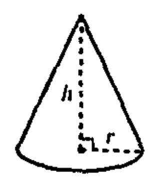
Cylinder

Volume: $V = \pi \cdot r^2 \cdot h$
Surface Area:
$A = 2\pi rh + 2\pi r^2$



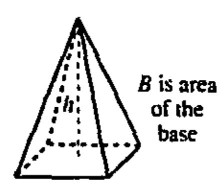
Cone

Volume: $V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$
Surface Area: $A = \pi r\sqrt{r^2 + h^2}$



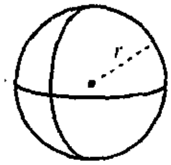
Pyramid

Volume: $V = \frac{1}{3} \cdot B \cdot h$



Sphere

Volume: $V = \frac{4}{3} \cdot \pi \cdot r^3$
Surface Area $= 4\pi r^2$



American - American Conversions

Distance			Volume		
12 inches (in)	=	1 foot (ft)	3 teaspoons (tsp)	=	1 tablespoon (tbsp)
3 feet	=	1 yard (yd)	1 tablespoon	=	0.5 fluid ounces
5280 feet	=	1 mile	8 fluid ounces (fl oz)	=	1 cup (c)
8 furlongs	=	1 mile	2 cups	=	1 pint (pt)
6 feet	=	1 fathom	2 pints	=	1 quart (qt)
100 links	=	1 chain	4 quarts	=	1 gallon (gal)
66 feet	=	1 chain	1 ft^3	≈	7.481 gallons

Area			Weight		
640 acres	=	1 mi^2	480 grains	=	1 ounce (oz)
			16 ounces	=	1 pound (lb)
			2000 pounds	=	1 ton

Metric - Metric Conversions

Distance			Volume		
1 kilometer (km)	=	1000 m	1000 milliliter (ml)	=	1 liter
100 centimeters (cm)	=	1 meter	1 cm^3	=	1 milliliter
1000 millimeters (mm)	=	1 meter			
10 decimeters (dm)	=	1 meter			

Area			Weight		
1 hectare	=	10,000 m^2	1 kilogram (kg)	=	1000 grams
			1000 milligrams (mg)	=	1 gram (g)
			1000 kilograms	=	1 metric ton

American - Metric Conversions

Distance			Volume			Weight		
2.54 cm	=	1 inch	1.06 qt	≈	1 liter	454 g	≈	1 lb
39.37 in	≈	1 meter				2.2 lb	≈	1 kg
1.609 km	≈	1 mile						

Time Conversions

60 seconds (sec)	=	1 minute (min)
60 minutes	=	1 hour (hr)
24 hours	=	1 day (da)
7 days	=	1 week (wk)
365 days	=	1 calendar year (non-leap year)
365.242199 days	=	1 tropical year

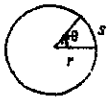
Metric Prefixes

Prefix	Multiple		Prefix	Fraction	
tera	10^{12}	T	deci	10^{-1}	d
giga	10^{9}	G	centi	10^{-2}	c
mega	10^{6}	M	milli	10^{-3}	m
kilo	10^{3}	k	micro	10^{-6}	µ
hecto	10^{2}	h	nano	10^{-9}	n
deca	10^{1}	da	pico	10^{-12}	p

TRIGONOMENTRY HELP SHEET

Definition of a Radian

One radian is the measure of a central angle θ th‥ subtends(intercepts) an arc s equal in length t‥ radius r of the circle.
Arc length= Radius when $\theta = 1$ radian



Conversion: Degrees ⟷ Radians

1. To convert degrees to radians multiply by $\dfrac{\pi \quad rad}{180^\circ}$

2. To convert radians to degrees multiply by $\dfrac{180^\circ}{\pi \quad rad}$

Uniform Circular Motion

A point on a circle of radius r moves a distance s on the circumference of the circle, in an amount of time t. θ *is measured in radians*

Angular Velocity: $\omega = \dfrac{\theta}{t}$

Arc Length: $s = r\theta$

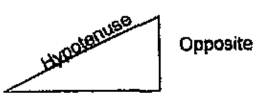
Linear Velocity: $v = \dfrac{s}{t}$

Area of a sector: $A = \dfrac{1}{2} r^2 \theta$

$v = r\omega$

Definition of the Six Trigonometric Functions

Right Triangle definitions, where $0<\theta<\pi/2$

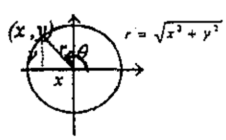


$\sin\theta = \dfrac{opp.}{hyp.}$ $\csc\theta = \dfrac{hyp.}{opp.}$

$\cos\theta = \dfrac{adj.}{hyp.}$ $\sec\theta = \dfrac{hyp.}{adj.}$

$\tan\theta = \dfrac{opp.}{adj.}$ $\cot\theta = \dfrac{adj.}{opp.}$

Circular function definitions, where θ is any angle



$r = \sqrt{x^2 + y^2}$

$\sin\theta = \dfrac{y}{r}$ $\csc\theta = \dfrac{r}{y}$

$\cos\theta = \dfrac{x}{r}$ $\sec\theta = \dfrac{r}{x}$

$\tan\theta = \dfrac{y}{x}$ $\cot\theta = \dfrac{x}{y}$

Quotient Identities

$\tan u = \dfrac{\sin u}{\cos u}$ $\cot u = \dfrac{\cos u}{\sin u}$

Reciprocal Identities

$\sin u = \dfrac{1}{\csc u}$ $\csc u = \dfrac{1}{\sin u}$

$\cos u = \dfrac{1}{\sec u}$ $\sec u = \dfrac{1}{\cos u}$

$\tan u = \dfrac{1}{\cot u}$ $\cot u = \dfrac{1}{\tan u}$

Pythagorean Identities

$\sin^2 u + \cos^2 u = 1$

$1 + \tan^2 u = \sec^2 u$

$1 + \cot^2 u = \csc^2 u$

Even/Odd Identities

$\sin(-u) = -\sin u$

$\cos(-u) = \cos u$

$\tan(-u) = -\tan u$

‥unction Identities

$\sin(\frac{\pi}{2} - u) = \cos u$

$\cos(\frac{\pi}{2} - u) = \sin u$

$\tan(\frac{\pi}{2} - u) = \cot u$

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u$$
$$= 2\cos^2 u - 1$$
$$= 1 - 2\sin^2 u$$

$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$$

Half-Angle Formulas

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$$

$$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

The signs of $\sin u/2$ *and* $\cos u/2$ *depend on the quadrant in which* $u/2$ *lies.*

Power-Reducing Formulas

$$\sin^2 u = \frac{1-\cos 2u}{2}$$

$$\cos^2 u = \frac{1+\cos 2u}{2}$$

$$\tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

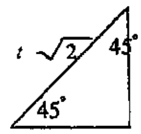
Product-to-Sum Formulas

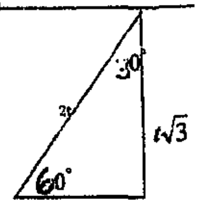
$$\sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$$

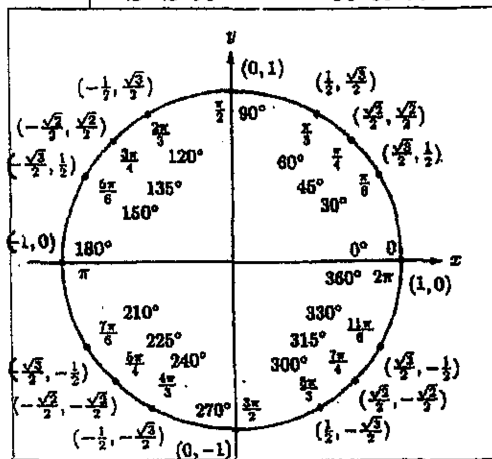
$$\cos u \cos v = \frac{1}{2}[\cos(u-v) + \cos(u+v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$$

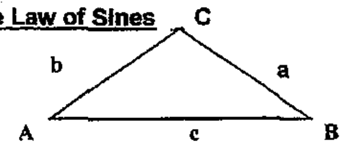
$$\cos u \sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$$







The Law of Sines



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The Law of Cosines

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

or

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Heron's Formula for Area of a triangle

If the semiperimeter is s, *where*

$$s = \frac{a+b+c}{2}$$

then $A = \sqrt{s(s-a)(s-b)(s-c)}$

Trig. Form of a complex Number:

$x + iy = r[\cos(\theta + 2n\pi) + i\sin(\theta + 2n\pi)]$,

n any integer

Graphs of Trigonometric and Inverse Trigonometric Functions:

Graphing Sine and Cosine Curves

The graphs of y =Asin(Bx+C)+D and y=Acos(Bx+C)+D, where B> 0, will have the following characteristics:

$$Amplitude = |A| \quad Period = \frac{2\pi}{B} \quad Frequency = \frac{B}{2\pi}$$

$$PhaseShift = \frac{-C}{B}\begin{cases} left \ if \ -C/B<0 \\ right \ if \ -C/B>0 \end{cases}$$

$$D = Vertical \ \ Translation$$

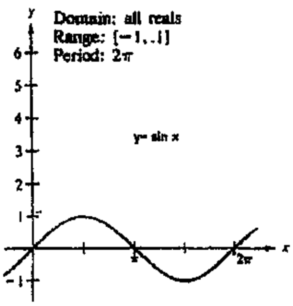
Graphing Tangent and Cotangent Curves

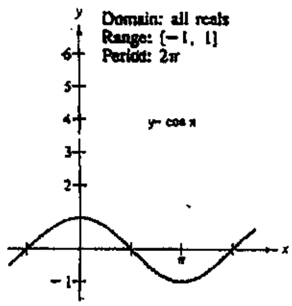
The graphs of y =Atan(Bx+C)+D and y=Acot(Bx+C)+D, where B> 0, will have the following characteristics:

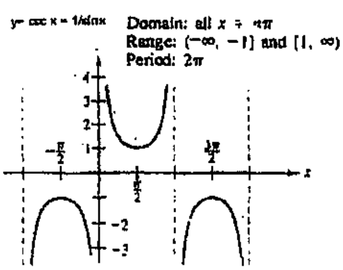
$$Amplitude = |A| \quad Period = \frac{\pi}{B} \quad Frequency = \frac{B}{\pi}$$

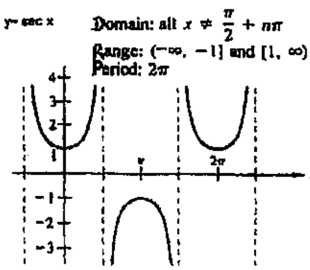
$$PhaseShift = \frac{-C}{B}\begin{cases} left \ if \ -C/B<0 \\ right \ if \ -C/B>0 \end{cases}$$

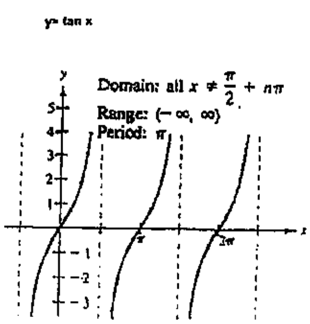
$$D = Vertical \ \ Translation$$

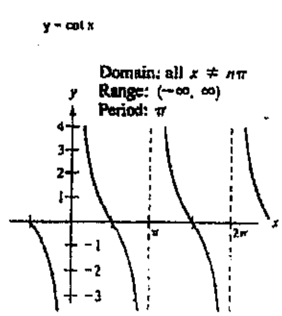












Inverse Trigonometric Functions:

Inverse Function	Meaning
$y=\sin^{-1}x$ or $y=\arcsin x$	$x=\sin y$ and $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$

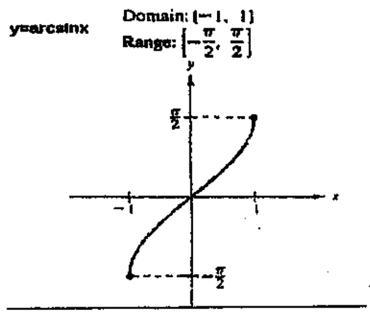
In words: *y is the angle between* $-\frac{\pi}{2}$ *and* $\frac{\pi}{2}$, *inclusive, whose* sine *is x.*

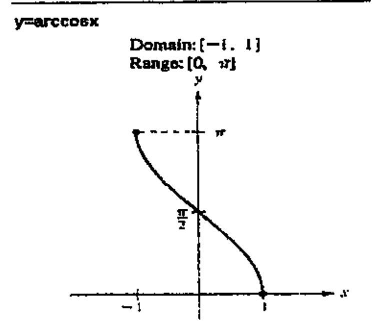
$y=\cos^{-1}x$ or $y=\arccos x$ $\qquad x=\cos y$ and $0\le y\le\pi$

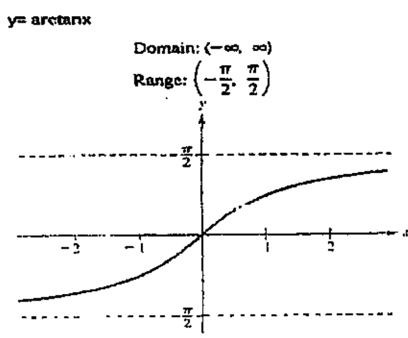
In words: *y is the angle between* 0 *and* π, *inclusive, whose* cosine *is x*

$y=\tan^{-1}x$ or $y=\arctan x$ $\qquad x=\tan y$ and $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$

In words: *y is the angle between* $-\frac{\pi}{2}$ *and* $\frac{\pi}{2}$, *whose* tangent *is x*







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