**Significant Figures**

It is important to be honest when reporting a measurement, so that it does not appear to be more accurate than the equipment used to make the measurement allows. We can achieve this by controlling the number of digits, or significant figures, used to report the measurement.

**Determining the Number of Significant Figures**

The number of significant figures in a measurement, such as 2.531, is equal to the number of digits that are known with some degree of confidence (2, 5, and 3) plus the last digit (1), which is an estimate or approximation. As we improve the sensitivity of the equipment used to make a measurement, the number of significant figures increases.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Example</th>
<th>Significant Figures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Postage scale</td>
<td>3 ± 1 g</td>
<td>1</td>
</tr>
<tr>
<td>Two-pan balance</td>
<td>2.53 ± 0.01 g</td>
<td>3</td>
</tr>
<tr>
<td>Analytical balance</td>
<td>2.531 ± 0.001 g</td>
<td>4</td>
</tr>
</tbody>
</table>

Rules for counting significant figures are summarized below.

Zeros within a number are always significant. Both 4308 and 40.05 contain four significant figures.

Zeros that do nothing but set the decimal point are not significant. Thus, 470,000 has two significant figures.

Trailing zeros that aren’t needed to hold the decimal point are significant. For example, 4.00 has three significant figures.

If you are not sure whether a digit is significant, assume that it isn’t. For example, if the directions for an experiment read: “Add the sample to 400 mL of water,” assume the volume of water is known to one significant figure.

**Addition and Subtraction with Significant Figures**

When combining measurements with different degrees of accuracy and precision, the accuracy of the final answer can be no greater than the least accurate measurement. This principle can be translated into a simple rule for addition and subtraction: When measurements are added or subtracted, the answer can contain no more decimal places than the least accurate measurement.

**Example:**

\[
150.0 \text{ g H}_2\text{O} \\
+ 0.507 \text{ g salt} \\
150.5 \text{ g solution}
\]
Multiplication and Division with Significant Figures

The same principle governs the use of significant figures in multiplication and division: the final result can be no more accurate than the least accurate measurement. In this case, however, we count the significant figures in each measurement, not the number of decimal places: When measurements are multiplied or divided, the answer can contain no more significant figures than the least accurate measurement.

Example:

Let's calculate the cost of copper in an old penny that is pure copper. Let's assume that the penny has a mass of 2.531 grams, that it is essentially pure copper, and that the price of copper is 67 cents per pound. We can start by converting from grams to pounds.

\[
2.531 \text{ g} \times \frac{1 \text{ lb}}{453.6 \text{ g}} = 0.005580 \text{ lb}
\]

We then use the price of a pound of copper to calculate the cost of the copper metal.

\[
0.005580 \text{ lb} \times \frac{67 \text{ c}}{1 \text{ lb}} = 0.3749 \text{ c}
\]

There are four significant figures in both the mass of the penny (2.531) and the number of grams in a pound (453.6). However, there are only two significant figures in the price of copper, so the final answer can only have two significant figures.

Rounding Off

When the answer to a calculation contains too many significant figures, it must be rounded off.

There are 10 digits that can occur in the last decimal place in a calculation. One way of rounding off involves underestimating the answer for five of these digits (0, 1, 2, 3, and 4) and overestimating the answer for the other five (5, 6, 7, 8, and 9). This approach to rounding off is summarized as follows.

If the digit is smaller than 5, drop this digit and leave the remaining number unchanged. Thus, 1.684 becomes 1.68.

If the digit is 5 or larger, drop this digit and add 1 to the preceding digit. Thus, 1.247 becomes 1.25.
GEOMETRIC FORMULAS

Rectangle
Perimeter: $P = 2 \cdot l + 2 \cdot w$
Area: $A = l \cdot w$

Right Triangle
Hypotenuse = $\sqrt{(leg)^2 + (leg)^2}$
Leg = $\sqrt{(hypothesis)^2 - (leg)^2}$

Square
Perimeter: $P = 4 \cdot s$
Area: $A = s^2$

Rectangular Solid
Volume: $V = l \cdot w \cdot h$
Surface Area: $A = 2HW + 2LW + 2LH$

Cube
Volume: $V = s^3$
Surface Area: $A = 6s^2$

Parallelogram
Perimeter: Add the lengths of the sides.
Area: $A = b \cdot h$

Cylinder
Volume: $V = \pi \cdot r^2 \cdot h$
Surface Area: $A = 2\pi rh + 2\pi r^2$

Triangle
Perimeter: Add the lengths of the sides.
Area: $A = \frac{1}{2} \cdot b \cdot h$

Cone
Volume: $V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$
Surface Area: $A = \pi r\sqrt{r^2 + h^2}$

Trapezoid
Perimeter: Add the lengths of the sides.
Area: $A = \frac{1}{2} \cdot h \cdot (b + B)$

Pyramid
Volume: $V = \frac{1}{3} \cdot B \cdot h$

Circle
Diameter: $d = 2 \cdot r$
Radius: $r = \frac{d}{2}$
Circumference: $C = \pi \cdot d$ or $C = 2 \cdot \pi \cdot r$
Area: $A = \pi \cdot r^2$
Use 3.14 as the approximate value of $\pi$.

Sphere
Volume: $V = \frac{4}{3} \cdot \pi \cdot r^3$
Surface Area = $4\pi r^2$
### American - American Conversions

<table>
<thead>
<tr>
<th>Conversion</th>
<th>Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 inches (in)</td>
<td>1 foot (ft)</td>
</tr>
<tr>
<td>3 feet</td>
<td>1 yard (yd)</td>
</tr>
<tr>
<td>5280 feet</td>
<td>1 mile</td>
</tr>
<tr>
<td>8 furlongs</td>
<td>1 mile</td>
</tr>
<tr>
<td>6 feet</td>
<td>1 fathom</td>
</tr>
<tr>
<td>100 links</td>
<td>1 chain</td>
</tr>
<tr>
<td>66 feet</td>
<td>1 chain</td>
</tr>
<tr>
<td>3 teaspoons (tsp)</td>
<td>1 tablespoon (tbsp)</td>
</tr>
<tr>
<td>1 tablespoon</td>
<td>0.5 fluid ounces</td>
</tr>
<tr>
<td>8 fluid ounces (fl oz)</td>
<td>1 cup (c)</td>
</tr>
<tr>
<td>2 cups</td>
<td>1 pint (pt)</td>
</tr>
<tr>
<td>2 pints</td>
<td>1 quart (qt)</td>
</tr>
<tr>
<td>4 quarts</td>
<td>1 gallon (gal)</td>
</tr>
<tr>
<td>1 ft³</td>
<td>7.481 gallons</td>
</tr>
</tbody>
</table>

### Metric - Metric Conversions

<table>
<thead>
<tr>
<th>Conversion</th>
<th>Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 kilometer (km)</td>
<td>1000 m</td>
</tr>
<tr>
<td>100 centimeters (cm)</td>
<td>1 meter</td>
</tr>
<tr>
<td>1000 millimeters (mm)</td>
<td>1 meter</td>
</tr>
<tr>
<td>10 decimeters (dm)</td>
<td>1 meter</td>
</tr>
<tr>
<td>1000 milliliters (ml)</td>
<td>1 liter</td>
</tr>
<tr>
<td>1 cm³</td>
<td>1 milliliter</td>
</tr>
<tr>
<td>1 hectare</td>
<td>10,000 m²</td>
</tr>
<tr>
<td>1 kilogram (kg)</td>
<td>1000 grams</td>
</tr>
<tr>
<td>1000 milligrams (mg)</td>
<td>1 gram (g)</td>
</tr>
<tr>
<td>1000 kilograms</td>
<td>1 metric ton</td>
</tr>
</tbody>
</table>

### American - Metric Conversions

<table>
<thead>
<tr>
<th>Conversion</th>
<th>Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.54 cm</td>
<td>1 inch</td>
</tr>
<tr>
<td>1.06 qt</td>
<td>1 liter</td>
</tr>
<tr>
<td>39.37 in</td>
<td>1 meter</td>
</tr>
<tr>
<td>1.609 km</td>
<td>1 mile</td>
</tr>
<tr>
<td>454 g</td>
<td>1 lb</td>
</tr>
<tr>
<td>2.2 lb</td>
<td>1 kg</td>
</tr>
</tbody>
</table>

### Time Conversions

<table>
<thead>
<tr>
<th>Unit (sec)</th>
<th>Equivalent (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>24</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>365</td>
<td>1</td>
</tr>
<tr>
<td>365.242199</td>
<td>1</td>
</tr>
</tbody>
</table>

### Metric Prefixes

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>tera</td>
<td>T</td>
<td>10¹²</td>
</tr>
<tr>
<td>giga</td>
<td>G</td>
<td>10⁹</td>
</tr>
<tr>
<td>mega</td>
<td>M</td>
<td>10⁶</td>
</tr>
<tr>
<td>kilo</td>
<td>k</td>
<td>10³</td>
</tr>
<tr>
<td>hecto</td>
<td>h</td>
<td>10²</td>
</tr>
<tr>
<td>deca</td>
<td>da</td>
<td>10¹</td>
</tr>
<tr>
<td>centi</td>
<td>c</td>
<td>10⁻²</td>
</tr>
<tr>
<td>milli</td>
<td>m</td>
<td>10⁻³</td>
</tr>
<tr>
<td>micro</td>
<td>μ</td>
<td>10⁻⁶</td>
</tr>
<tr>
<td>nano</td>
<td>n</td>
<td>10⁻⁹</td>
</tr>
<tr>
<td>pico</td>
<td>p</td>
<td>10⁻¹²</td>
</tr>
</tbody>
</table>
Trigonometry Help Sheet

**Definition of a Radian**

One radian is the measure of a central angle \( \theta \) that subtends an arc \( s \) equal in length to the radius \( r \) of the circle.

Arc length: \( r \theta \) when \( \theta \) is measured in radians

**Conversion: Degrees \( \rightleftharpoons \) Radians**

1. To convert degrees to radians \( \frac{\pi}{180} \times \text{degrees} \)
2. To convert radians to degrees \( \frac{180}{\pi} \times \text{radians} \)

**Uniform Circular Motion**

A point on a circle of radius \( r \) moves a distance \( s \) on the circumference of the circle, in an amount of time \( t \) is measured in radians.

Angular Velocity: \( \omega = \frac{s}{t} \)

Linear Velocity: \( v = \frac{d}{t} = \frac{1}{2} r \omega \)

**Definition of the Six Trigonometric Functions**

Right Triangle definitions, where \( 0 < \theta < \frac{\pi}{2} \)

- **Hypotenuse**: \( \sin \theta = \frac{opp}{hyp} \) \( \cos \theta = \frac{adj}{hyp} \) \( \tan \theta = \frac{opp}{adj} \)
- **Adjacent**: \( \sin \theta = \frac{YP}{X} \) \( \cos \theta = \frac{X}{YP} \) \( \tan \theta = \frac{YP}{X} \)
- **Circular function definitions, where \( \theta \) is any angle**: \( \cos \theta = \frac{YP}{X} \) \( \sin \theta = \frac{X}{YP} \) \( \tan \theta = \frac{YP}{X} \)

**Quotient Identities**: \( \tan \theta = \frac{\sin \theta}{\cos \theta} \) \( \cot \theta = \frac{\cos \theta}{\sin \theta} \)

**Pythagorean Identities**: \( \sin^2 \theta + \cos^2 \theta = 1 \) \( 1 + \tan^2 \theta = \sec^2 \theta \) \( 1 + \cot^2 \theta = \csc^2 \theta \)

**Sum and Difference Formulas**

\[
\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \\
\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \\
\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}
\]

**Double-Angle Formulas**

\[
\begin{align*}
\sin 2\theta &= 2 \sin \theta \cos \theta \\
\cos 2\theta &= \cos^2 \theta - \sin^2 \theta = 1 - 2 \sin^2 \theta = 2 \cos^2 \theta - 1
\end{align*}
\]

**Half-Angle Formulas**

\[
\begin{align*}
\sin \frac{\theta}{2} &= \pm \sqrt{\frac{1 - \cos \theta}{2}} \\
\cos \frac{\theta}{2} &= \pm \sqrt{\frac{1 + \cos \theta}{2}} \\
\tan \frac{\theta}{2} &= \frac{1 - \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 + \cos \theta}
\end{align*}
\]

The signs of \( \sin \theta /2 \) and \( \cos \theta /2 \) depend on the quadrant in which \( \theta /2 \) lies.

**Power-Reducing Formulas**

\[
\begin{align*}
\sin^2 \theta &= \frac{1 - \cos 2\theta}{2} \\
\cos^2 \theta &= \frac{1 + \cos 2\theta}{2} \\
\tan^2 \theta &= \frac{1 - \cos 2\theta}{1 + \cos 2\theta}
\end{align*}
\]

**Sum-to-Product Formulas**

\[
\begin{align*}
\sin \alpha + \sin \beta &= 2 \sin \left( \frac{\alpha + \beta}{2} \right) \cos \left( \frac{\alpha - \beta}{2} \right) \\
\sin \alpha - \sin \beta &= 2 \cos \left( \frac{\alpha + \beta}{2} \right) \sin \left( \frac{\alpha - \beta}{2} \right) \\
\cos \alpha + \cos \beta &= 2 \cos \left( \frac{\alpha + \beta}{2} \right) \cos \left( \frac{\alpha - \beta}{2} \right) \\
\cos \alpha - \cos \beta &= -2 \sin \left( \frac{\alpha + \beta}{2} \right) \sin \left( \frac{\alpha - \beta}{2} \right)
\end{align*}
\]

**Product-to-Sum Formulas**

\[
\begin{align*}
\sin \alpha \sin \beta &= \frac{1}{2} \left[ \cos(\alpha - \beta) - \cos(\alpha + \beta) \right] \\
\cos \alpha \cos \beta &= \frac{1}{2} \left[ \cos(\alpha - \beta) + \cos(\alpha + \beta) \right] \\
\sin \alpha \cos \beta &= \frac{1}{2} \left[ \sin(\alpha + \beta) + \sin(\alpha - \beta) \right] \\
\cos \alpha \sin \beta &= \frac{1}{2} \left[ \sin(\alpha + \beta) - \sin(\alpha - \beta) \right]
\end{align*}
\]

**The Law of Sines**

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

**The Law of Cosines**

\[
a^2 = b^2 + c^2 - 2bc \cos A \\
b^2 = c^2 + a^2 - 2ac \cos B \\
c^2 = a^2 + b^2 - 2ab \cos C
\]

**Heron’s Formula for Area of a Triangle**

If the semiperimeter is \( s \), where

\[ s = \frac{a + b + c}{2} \]

then

\[ A = \sqrt{s(s-a)(s-b)(s-c)} \]

**Trig. Form of a complex number:**

\[ x + iy = r(\cos \theta + 2\pi n) + i(\sin \theta + 2\pi n) \]

\( n \) any integer
Graphs of Trigonometric and Inverse Trigonometric Functions:

Graphing Sine and Cosine Curves
The graphs of \( y = \sin(Bx+C) + D \) and \( y = \cos(Bx+C) + D \), where \( B > 0 \), will have the following characteristics:

Amplitude = \( |A| \)  
Period = \( \frac{2\pi}{B} \)  
Frequency = \( \frac{B}{2\pi} \)

PhaseShift = \( -\frac{C}{B} \) left if \( -\frac{C}{B} < 0 \), right if \( -\frac{C}{B} > 0 \)  
\( D = \text{Vertical Translation} \)

Graphing Tangent and Cotangent Curves
The graphs of \( y = \tan(Bx+C) + D \) and \( y = \cot(Bx+C) + D \), where \( B > 0 \), will have the following characteristics:

Amplitude = \( |A| \)  
Period = \( \frac{\pi}{B} \)  
Frequency = \( \frac{B}{\pi} \)

PhaseShift = \( -\frac{C}{B} \) left if \( -\frac{C}{B} < 0 \), right if \( -\frac{C}{B} > 0 \)  
\( D = \text{Vertical Translation} \)

Inverse Trigonometric Functions:

<table>
<thead>
<tr>
<th>Inverse Function</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = \sin^{-1} x ) or ( y = \arcsin x )</td>
<td>( x = \sin y ) and ( -\frac{\pi}{2} \leq y \leq \frac{\pi}{2} )</td>
</tr>
</tbody>
</table>

In words: \( y \) is the angle between \( -\frac{\pi}{2} \) and \( \frac{\pi}{2} \), inclusive, whose sine is \( x \).

<table>
<thead>
<tr>
<th>Inverse Function</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = \cos^{-1} x ) or ( y = \arccos x )</td>
<td>( x = \cos y ) and ( 0 \leq y \leq \pi )</td>
</tr>
</tbody>
</table>

In words: \( y \) is the angle between \( 0 \) and \( \pi \), inclusive, whose cosine is \( x \).

<table>
<thead>
<tr>
<th>Inverse Function</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = \tan^{-1} x ) or ( y = \arctan x )</td>
<td>( x = \tan y ) and ( -\frac{\pi}{2} \leq y \leq \frac{\pi}{2} )</td>
</tr>
</tbody>
</table>

In words: \( y \) is the angle between \( -\frac{\pi}{2} \) and \( \frac{\pi}{2} \), whose tangent is \( x \).

---

Suba Marti, Foothill College Math Center.