Differentiation Formulas

1. $\left(f(x) \pm g(x)\right)' = f'(x) \pm g'(x)$

2. $\left(kf(x)\right)' = kf'(x)$

3. $\left(f(x)g(x)\right)' = f(x)g'(x) + g(x)f'(x)$

4. $\left(\dfrac{f(x)}{g(x)}\right)' = \dfrac{g(x)f'(x) - f(x)g'(x)}{\left(g(x)\right)^2}$

5. $\left(f\left(g(x)\right)\right)' = f'\left(g(x)\right) \cdot g'(x)$

6. $\dfrac{d}{dx}(x^n) = nx^{n-1}$

7. $\dfrac{d}{dx}(e^x) = e^x$

8. $\dfrac{d}{dx}(a^x) = a^x \ln a \quad (a > 0)$

9. $\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$

10. $\dfrac{d}{dx}(\sin x) = \cos x$

11. $\dfrac{d}{dx}(\cos x) = -\sin x$

12. $\dfrac{d}{dx}(\tan x) = \dfrac{1}{\cos^2 x}$

A Short Table of Indefinite Integrals

Basic Functions

1. $\displaystyle\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$

2. $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$

3. $\displaystyle\int a^x\,dx = \frac{1}{\ln a}a^x + C, \quad a > 0$

4. $\displaystyle\int \ln x\,dx = x\ln x - x + C$

5. $\displaystyle\int \sin x\,dx = -\cos x + C$

6. $\displaystyle\int \cos x\,dx = \sin x + C$

7. $\displaystyle\int \tan x\,dx = -\ln|\cos x| + C$

I. Products of e^x, $\cos x$, and $\sin x$

8. $\displaystyle\int e^{ax}\sin(bx)\,dx = \frac{1}{a^2+b^2}e^{ax}[a\sin(bx) - b\cos(bx)] + C$

9. $\displaystyle\int e^{ax}\cos(bx)\,dx = \frac{1}{a^2+b^2}e^{ax}[a\cos(bx) + b\sin(bx)] + C$

10. $\displaystyle\int \sin(ax)\sin(bx)\,dx = \frac{1}{b^2-a^2}[a\cos(ax)\sin(bx) - b\sin(ax)\cos(bx)] + C, \quad a \neq b$

11. $\displaystyle\int \cos(ax)\cos(bx)\,dx = \frac{1}{b^2-a^2}[b\cos(ax)\sin(bx) - a\sin(ax)\cos(bx)] + C, \quad a \neq b$

12. $\displaystyle\int \sin(ax)\cos(bx)\,dx = \frac{1}{b^2-a^2}[b\sin(ax)\sin(bx) + a\cos(ax)\cos(bx)] + C, \quad a \neq b$

III. Product of Polynomial $p(x)$ with $\ln x$, e^x, $\cos x$, $\sin x$

13. $\displaystyle\int x^n\ln x\,dx = \frac{1}{n+1}x^{n+1}\ln x - \frac{1}{(n+1)^2}x^{n+1} + C, \quad n \neq -1$

14. $$\int p(x)e^{ax}\,dx = \frac{1}{a}p(x)e^{ax} - \frac{1}{a}\int p'(x)e^{ax}\,dx$$
$$= \frac{1}{a}p(x)e^{ax} - \frac{1}{a^2}p'(x)e^{ax} + \frac{1}{a^3}p''(x)e^{ax} - \cdots$$
$(+ - + - \ldots)$

(signs alternate)

15. $$\int p(x)\sin ax\,dx = -\frac{1}{a}p(x)\cos ax + \frac{1}{a}\int p'(x)\cos ax\,dx$$
$$= -\frac{1}{a}p(x)\cos ax + \frac{1}{a^2}p'(x)\sin ax + \frac{1}{a^3}p''(x)\cos ax - \cdots$$
$(- + + - - + + \ldots)$

(signs alternate in pairs after first term)

16. $$\int p(x)\cos ax\,dx = \frac{1}{a}p(x)\sin ax - \frac{1}{a}\int p'(x)\sin ax\,dx$$
$$= \frac{1}{a}p(x)\sin ax + \frac{1}{a^2}p'(x)\cos ax - \frac{1}{a^3}p''(x)\sin ax - \cdots$$
$(+ + - - + + - - \ldots)$ (signs alternate in pairs)

IV. Integer Powers of $\sin x$ and $\cos x$

17. $$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx, \quad n \text{ positive}$$

18. $$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx, \quad n \text{ positive}$$

19. $$\int \frac{1}{\sin^m x}\,dx = \frac{-1}{m-1}\frac{\cos x}{\sin^{m-1}x} + \frac{m-2}{m-1}\int \frac{1}{\sin^{m-2}x}\,dx, \quad m \neq 1, m \text{ positive}$$

20. $$\int \frac{1}{\sin x}\,dx = \frac{1}{2}\ln\left|\frac{(\cos x)-1}{(\cos x)+1}\right| + C$$

21. $$\int \frac{1}{\cos^m x}\,dx = \frac{1}{m-1}\frac{\sin x}{\cos^{m-1}x} + \frac{m-2}{m-1}\int \frac{1}{\cos^{m-2}x}\,dx, \quad m \neq 1, m \text{ positive}$$

22. $$\int \frac{1}{\cos x}\,dx = \frac{1}{2}\ln\left|\frac{(\sin x)+1}{(\sin x)-1}\right| + C$$

23. $\int \sin^m x\cos^n x\,dx$: If m is odd, let $w = \cos x$. If n is odd, let $w = \sin x$. If both m and n are even and non-negative, convert all to $\sin x$ or all to $\cos x$ (using $\sin^2 x + \cos^2 x = 1$), and use IV-17 or IV-18. If m and n are even and one of them is negative, convert to whichever function is in the denominator and use IV-19 or IV-21. If both m and n are even and negative, the substitution $w = \cos x$ converts the integral into a rational function which can be integrated by the method of partial fractions.

V. Quadratic in the Denominator

24. $$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + C, \quad a \neq 0$$

25. $$\int \frac{bx+c}{x^2+a^2}\,dx = \frac{b}{2}\ln|x^2+a^2| + \frac{c}{a}\arctan\frac{x}{a} + C, \quad a \neq 0$$

26. $$\int \frac{1}{(x-a)(x-b)}\,dx = \frac{1}{a-b}(\ln|x-a| - \ln|x-b|) + C, \quad a \neq b$$

27. $$\int \frac{cx+d}{(x-a)(x-b)}\,dx = \frac{1}{a-b}\left[(ac+d)\ln|x-a| - (bc+d)\ln|x-b|\right] + C, \quad a \neq b$$

VI. Integrands Involving $\sqrt{a^2+x^2}$, $\sqrt{a^2-x^2}$, $\sqrt{x^2-a^2}$, $a > 0$

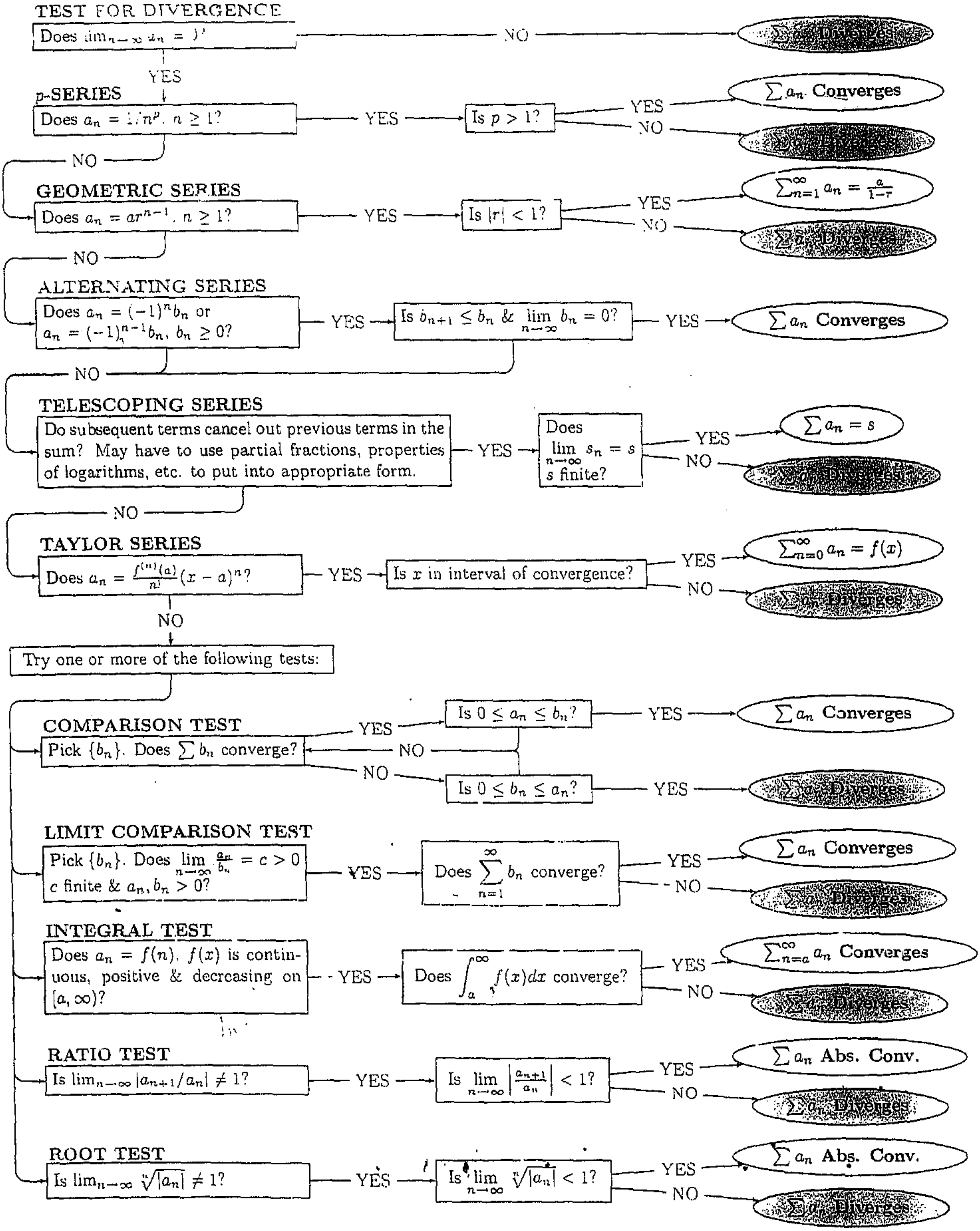
28. $$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\frac{x}{a} + C$$

29. $$\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$$

30. $$\int \sqrt{a^2 \pm x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2 \pm x^2} + a^2\int \frac{1}{\sqrt{a^2 \pm x^2}}\,dx\right) + C$$

31. $$\int \sqrt{x^2 - a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2-a^2} - a^2\int \frac{1}{\sqrt{x^2-a^2}}\,dx\right) + C$$

Series Convergence/Divergence Flow Chart



Some Important Maclaurin Series

Remark: Make sure that you can derive ANY of the examples below by taking derivatives of $f(x)$

FUNCTION	TAYLOR SERIES ABOUT x = 0	INTERVAL OF CONVERGENCE
$\sin x$	$x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\frac{x^9}{9!}-\cdots$ $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$	$-\infty<x<+\infty$
$\cos x$	$1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\frac{x^8}{8!}-\cdots$ $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$	$-\infty<x<+\infty$
e^x	$1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\cdots$ $\sum_{n=0}^{\infty}\frac{x^n}{n!}$	$-\infty<x<+\infty$
$f(x)$	$f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\cdots+\frac{f^{(n)}(0)}{n!}x^n+\cdots$ $\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$	Use Ratio Test to find the interval of convergence and always test the endpoints.
$\ln(1+x)$	$x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots=\sum_{n=0}^{\infty}(-1)^n\frac{x^{n+1}}{n+1}$	$-1<x\le 1$
$\arctan x$	$x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$ $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$	$-1\le x\le 1$
$(1+x)^p$	$1+px+\frac{p(p-1)}{2!}x^2+\frac{p(p-1)(p-2)}{3!}x^3+\cdots=\sum_{n=0}^{\infty}{}_pC_n x^n$	or $\sum_{n=0}^{\infty}\binom{k}{n}x^n$ $-1<x<1$
	SOME COMMON SPECIAL CASES OF THE BINOMIAL SERIES	${}_pC_n=\frac{p(p-1)\cdots(p-n+1)}{n!}$
$\frac{1}{1+x}$; $p=-1$	$1-x+x^2-x^3+x^4-\cdots$ $\sum_{n=0}^{\infty}(-1)^n x^n$	$-1<x<1$
$\frac{1}{1-x}$ $p=-1$	$1+x+x^2+x^3+x^4+\cdots$ $\sum_{n=0}^{\infty}x^n$	$-1<x<1$
$\frac{1}{(1+x)^2}$; $p=-2$	$1-2x+3x^2-4x^3+5x^4-\cdots=\sum_{n=0}^{\infty}(-1)^n(n+1)x^n$	$-1<x<1$
$\frac{1}{(1-x)^2}=\frac{1}{(x-1)^2}$	$1+2x+3x^2+4x^3+\cdots=\sum_{n=0}^{\infty}(n+1)x^n$	$-1<x<1$

$(1+x)^{1/2}_{p=1/2}=1+\frac{x}{2}-\frac{1}{2^2}\cdot\frac{x^2}{2!}+\frac{1\cdot3\cdot x^3}{2^3\,3!}-\frac{1\cdot3\cdot5\,x^4}{2^4\,4!}\cdots$ $\quad -1<x<1$

$=1+\frac{x}{2}+\sum_{n=2}^{\infty}(-1)^{n-1}\frac{1\cdot3\cdot5\cdots(2n-3)}{2^n\,n!}x^n$

general Taylor Series: $f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$