Algebra Help Sheet

Sets and The Real Number System

Natural Numbers: (counting numbers)
1, 2, 3, 4, 5, 6, ...

Prime Numbers:
(only divisible by itself & 1)
2, 3, 5, 7, 11, 13, 17, ...

Composite Numbers:
(not prime, divisible by more than 1 and itself)
4, 6, 8, 9, 10, 12, 14, ...

Whole Numbers: (positive numbers, no decimal or fraction)
0, 1, 2, 3, 4, 5, 6, ...

Integers: (negative and positive numbers)
... -3, -2, -1, 0, 1, 2, 3, ...

Rational Numbers:
Any number that can be written in the form:
\[ \frac{a}{b}, \quad (b \neq 0) \]
where \( a \) and \( b \) are integers.

Irrational Numbers:
Any number that is not rational, for example:
\[ \pi = 3.141592 \ldots \]
\[ e = 2.718281 \ldots \]
\[ \sqrt{2} = 1.414213 \ldots \]
cannot be written as \( \frac{a}{b} \) (where \( a \) and \( b \) are integers)

Real Numbers: All rational and irrational numbers.

Exponent Rules
If \( m \) and \( n \) are integers, and there is no division by 0, then:
\[ a^m \cdot a^n = a^{m+n} \]
\[ \frac{a^m}{a^n} = a^{m-n} \]
\[ (a^m)^n = a^{mn} \]
\[ a^{-n} = \frac{1}{a^n} \]
\[ (ab)^n = a^n b^n \]
\[ (ab)^{m/n} = (\sqrt[n]{a}) \]

Radical Rules
If \( n \) is a positive integer (> 1) and all radicals represent real numbers, then:
\[ \sqrt[n]{a} = a^{1/n} \]
\[ a^{m/n} = (\sqrt[n]{a})^m \]
\[ \sqrt[n]{a^m} = a \quad \text{if } n \text{ is even} \]
\[ \sqrt[n]{a^m} = a \quad \text{if } n \text{ is odd} \]
\[ \sqrt[ab]{b} = \frac{\sqrt[n]{b}}{\sqrt[n]{b}} \quad \text{if } b = 0 \]

Application of Radicals
Pythagorean Theorem:
If \( a \) and \( b \) are the lengths of two legs of a triangle and \( c \) is the length of the hypotenuse, then
\[ a^2 + b^2 = c^2 \]
The hypotenuse is: \( c = \sqrt{a^2 + b^2} \)

Distance Formula:
The distance between points \( P_1(x_1, y_1) \) and \( P_2(x_2, y_2) \) is given by the formula
\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

Quadratic Formula:
If \( p(x) = ax^2 + bx + c \) where \( a \neq 0 \) and \( b^2 - 4ac \geq 0 \), then
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
and the roots of \( p(x) = 0 \) are
\[ x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \]
\[ x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \]

Multiplying Polynomials: (use F.O.I.L.)
\[ a + c = a + c \]
\[ a - c = a - c \]
\[ (a + c) + (a - c) = 2a \]
\[ (a + c) - (a - c) = 2c \]
\[ a + c = a + c \]
\[ a - c = a - c \]
\[ (a + c)(a - c) = a^2 - c^2 \]
\[ (a + c) + (a - c) = 2a \]
\[ (a + c)(b + d) = (a + b)(c + d) \]
\[ (a + b)(c + d) = ac + ad + bc + bd \]
\[ (a + b)(c - d) = ac + ad - bc - bd \]
\[ (a - b)(c + d) = ac - ad + bc - bd \]
\[ (a - b)(c - d) = ac - ad - bc + bd \]
\[ a^2 + b^2 = (a + b)^2 \]
\[ a^2 - b^2 = (a + b)(a - b) \]
\[ a^2 \pm b^2 = (a + bi)(a - bi) \]
\[ a^2 - b^2 = (a + b)(a - b) \]
\[ a^2 + b^2 = (a + bi)(a - bi) \]

Solving Equations by Factoring
Zero-factor Theorem: Let \( a \) and \( b \) be real numbers, then:
If \( ab = 0 \), then \( a = 0 \) or \( b = 0 \).

Absolute Value Equations
If \( k \geq 0 \), then
\[ |x| = k \text{ is equivalent to } x = k \text{ or } x = -k \]
\[ |x| = k \text{ is equivalent to } a = b \text{ or } a = -b \]

Linear Inequalities
If \( a, b, c \) are real numbers and \( a < b \),
then \( a + c < b + c \) and \( a - c < b - c \)
\[ ac < bc \quad \text{if } c > 0 \quad \text{and } ac < bc \quad \text{if } c < 0 \]
\[ a < b \quad \text{if } c > 0 \quad \text{and } ac > bc \quad \text{if } c < 0 \]
\[ c < x < d \text{ is equivalent to } c < x \text{ and } x < d \]

Inequalities with Absolute Values
If \( k > 0 \), then
\[ |x| < k \text{ is equivalent to } -k < x < k \]
\[ |x| > k \text{ is equivalent to } x < -k \text{ or } x > k \]

Logarithmic Functions
\[ y = \log_{a} x \quad \text{if and only if} \quad x = a^y \]
\( f(x) = y = \log_{a} x \) and \( f^{-1}(x) = x = a^y \) are inverse functions

Properties of Logarithms
\[ \log_{a} b \times \log_{a} c = \log_{a} (bc) \]
\[ \log_{a} b \div \log_{a} c = \log_{a} \left( \frac{b}{c} \right) \]
\[ \log_{a} b^c = c \log_{a} b \]
\[ \log_{a} b + \log_{a} c = \log_{a} (bc) \]
\[ \log_{a} b - \log_{a} c = \log_{a} (\frac{b}{c}) \]
\[ \log_{a} b = \frac{1}{\log_{b} a} \]

Exponential Equation
\[ y = ax \]

Slope of a Line
Slope, \( m \), of the line containing the points \( (x_1, y_1) \) and \( (x_2, y_2) \)
\[ \text{slope, } m = y_2 - y_1 \]
\[ \text{horizontal lines have a slope of } 0 \]

Equations of Lines
Slope Intercept Form: \( y = mx + b \)
where \( m \) is the slope and \( b \) is the y-intercept

Point-Slope Form: \( y - y_1 = m(x - x_1) \)
where \( (x_1, y_1) \) is a point on the line.

General Form: \( Ax + By = C \)
Equation of a horizontal line: \( y = a \)
Equation of a vertical line: \( x = b \)

Composition Functions
\[ f \circ g(x) = f(g(x)) \]

Solving Equations by Factoring
Zero-factor Theorem: Let \( a \) and \( b \) be real numbers, then:
If \( ab = 0 \), then \( a = 0 \) or \( b = 0 \).