Algebra Help Sheet

Sets and The Real Number System

Natural Numbers:
(counting numbers)
1,2,3,4,5,6,...

Prime Numbers:
(only divisible by itself & 1)
2,3,4,7,11,13,17,...

Composite Numbers:
(not prime, divisible by more than 1 and itself)
4,6,8,9,10,12,14,...

Whole Numbers: (positive numbers, no decimal or fraction)
0,1,2,3,4,5,6,...

Integers: (negative and positive numbers)
...,-3,-2,-1,0,1,2,3, ...

Rational Numbers:
Any number that can be written in the form:

$$\frac{a}{b}, \quad (b \neq 0)$$

where a and b are integers.

Irrational Numbers:
Any number that is not rational, for example:

$$\pi = 3.141592\ldots$$
$$e = 2.718281\ldots$$
$$\sqrt{2} = 1.414213\ldots$$

cannot be written as $\frac{a}{b}$ (where a and b are integers)

Real Numbers: All rational and irrational numbers.

Equality and Properties of Real Numbers

If ***a***, ***b***, and ***c*** are real numbers, then
Reflexive property: $a = a$
Symmetric property: If $a = b$, then $b = a$
Transitive property: If $a = b$ and $b = c$, then $a = c$

Closure properties: $\begin{cases} a + b \text{ is a real number} \\ a - b \text{ is a real number} \\ a \cdot b \text{ is a real number} \\ a/b \text{ is a real number} \end{cases}$

Commutative property: $\begin{cases} a + b = b + a \\ a \cdot b = b \cdot a \end{cases}$

Associative property: $\begin{cases} (a + b) + c = a + (b + c) \\ (a \cdot b) \cdot c = a \cdot (b \cdot c) \end{cases}$

Distributive property: $a \cdot (b + c) = a \cdot b + a \cdot c$

0 is the additive identity: $a + 0 = a$
1 is the multiplicative identity: $a \cdot 1 = a$
a and $-a$ are additive inverses
a and $\frac{1}{a}$ are multiplicative inverses $(a \neq 0)$
Double negative rule: $-(-a) = a$

Arithmetic of Real Numbers

$a - b = a + (-b)$

If no denominator is 0, then:
$\frac{a}{b} = \frac{c}{d}$ if and only if $a \cdot d = b \cdot c$
$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ and $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$
$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$ and $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$

$\frac{a}{b} + \frac{c}{d} = \frac{ad+cb}{bd}$ and $\frac{a}{b} - \frac{c}{d} = \frac{ad-cb}{bd}$

$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$

Exponent Rules

If **m** and **n** are integers, and there is no division by 0, then:
$a^0 = 1$ and $a^1 = a$ and $a^{-1} = \frac{1}{a}$ and $a^{-n} = \frac{1}{a^n}$
$(ab)^n = a^n b^n$ and $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ and $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
$a^m a^n = a^{m+n}$ and $\frac{a^m}{a^n} = a^{m-n}$
$(a^m)^n = a^{m \cdot n}$

Radical Rules

If n is a positive integer $(n > 1)$ and all radicals represent real numbers, then:
$\sqrt[n]{a} = a^{1/n}$
$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
$\sqrt[n]{a^n} = |a|$ if **n** is even
$\sqrt[n]{a^n} = a$ if **n** is odd
$\sqrt{ab} = \sqrt{a}\sqrt{b}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $(b \neq 0)$

Application of Radicals

Pythagorean Theorem:
If **a** and **b** are the lengths of two legs of a triangle and **c** is the length of the hypotenuse, then

$$a^2 + b^2 = c^2$$

The hypotenuse is: $c = \sqrt{a^2 + b^2}$

Distance Formula:
The distance between points $P(x_1, y_1)$ and $P(x_2, y_2)$ is given by the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Quadratic Formula

If $p(x) = ax^2 + bx + c$ where $a \neq 0$ and $b^2 - 4ac \geq 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If	then the roots of $p(x) = 0$ are
$b^2 - 4ac > 0$	real and unequal
$b^2 - 4ac = 0$	rational and equal
$b^2 - 4ac < 0$	nonreal

If r_1 and r_2 are the roots of $ax^2 + bx + c = 0$

$$r_1 + r_2 = -\frac{b}{a} \text{ and } r_1 r_2 = \frac{c}{a}$$

Completing the square:
Add $\left(\frac{b}{2a}\right)^2$ to $x^2 + bx$ to complete the square:

$$x^2 + bx + \left(\frac{b}{2a}\right)^2 = \left(x + \frac{b}{a}\right)^2$$

Multiplying Polynomials: (use F.O.I.L.)

$a(b + c + d + \ldots) = ab + ac + ad + \ldots$
$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$
$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$
$(a - b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$

Factoring Polynomials

$ax + bx = x(a + b)$
$x(a + b) + y(a + b) = (x + y)(a + b)$
$a^2 - b^2 = (a + b)(a - b)$
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$a^4 - b^4 = (a - b)(a + b)(a^2 + b^2)$
$a^4 + b^4 = (a^2 + \sqrt{2}ab + b^2)(a^2 - \sqrt{2}ab + b^2)$

Solving Equations by Factoring

Zero-factor Theorem: Let **a** and **b** be real numbers, then:
If $ab = 0$, then $a = 0$ or $b = 0$.

Absolute Value Equations

If $k \geq 0$, then
$|x| = k$ is equivalent to $x = k$ or $x = -k$
$|a| = |b|$ is equivalent to $a = b$ or $a = -b$

Linear Inequalities

If a, b, and c are real numbers and $a < b$,

then $a + c < b + c$ and $a - c < b - c$
$ac < bc$ $(c > 0)$ and $ac < bc$ $(c < 0)$
$\frac{a}{c} < \frac{b}{c}$ $(c > 0)$ and $ac > bc$ $(c < 0)$
$c < x < d$ is equivalent to $c < x$ and $x < d$

Inequalities with Absolute Values

If $k > 0$, then
$|x| < k$ is equivalent to $-k < x < k$
$|x| > k$ is equivalent to $x < -k$ or $x > k$

Logarithmic Functions

$y = \log_b x$ if and only if $x = b^y$ $(b > 0 \text{ and real})$
$f(x) = y = \log_b x$ and $f^{-1}(x) = x = b^y$ are inverse functions

Properties of Logarithms

$\log_b 1 = 0$ $\qquad$ $\log_b b = 1$
$b^{\log_b x} = x$ $\qquad$ $\log_b b^x = x$
$\log_b MN = \log_b M + \log_b N$
$\log_b \frac{M}{N} = \log_b M - \log_b N$
$\log_b M^p = p \log_b M$
If $\log_b x = \log_b y$ then $x = y$

Change of base formula: $\log_b x = \frac{\log_a x}{\log_a b}$

Exponential Equation

$$y = ab^t$$

Slope of a Line

Slope, m, of the line containing the points (x_1, y_1) and (x_2, y_2)

$$Slope = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Horizontal lines have a slope of 0.
Vertical lines have no defined slope.
Lines with same slopes are parallel.
Lines with slopes that are negative reciprocals are perpendicular.

Equations of Lines

Slope Intercept Form: $y = mx + b$
where m is the slope and b is the y-intercept

Point-Slope Form: $y - y_1 = m(x - x_1)$
where (x_1, y_1) is a point on the line.

General Form: $Ax + By = C$

Equation of a horizontal line: $y = a$
Equation of a vertical line: $x = b$
where a and b are constant values

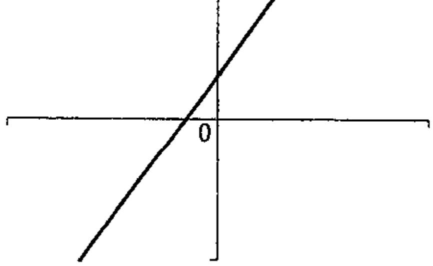
Composition Functions

$$(f \circ g)(x) = f(g(x))$$

LIBRARY OF FUNCTIONS SUMMARY

Linear Function

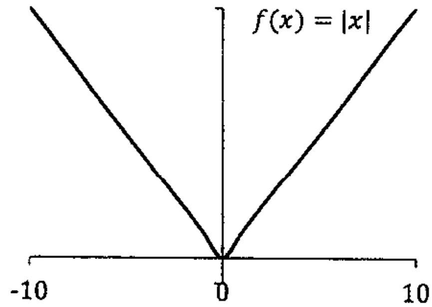
$f(x) = mx + b$



Domain: $(-\infty, \infty)$ **Range:** $(-\infty, \infty)$
x-intercept: $(-b/m, 0)$ **y-intercept:** $(0, b)$

Absolute Value Function

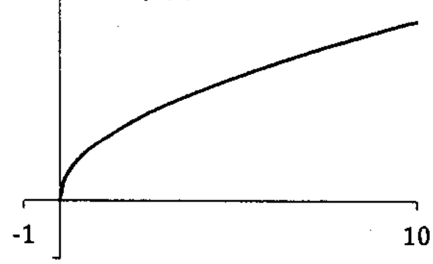
$f(x) = |x|$



Domain: $(-\infty, \infty)$ **Range:** $(0, \infty)$
x-intercept: $(0, 0)$ **y-intercept:** $(0, 0)$

Square Root Function

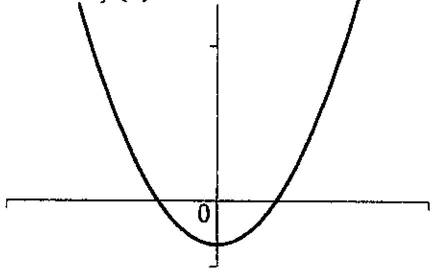
$f(x) = \sqrt{x}$



Domain: $(-\infty, \infty)$ **Range:** $(0, \infty)$
x-intercept: $(0, 0)$ **y-intercept:** $(0, 0)$

Quadratic Function

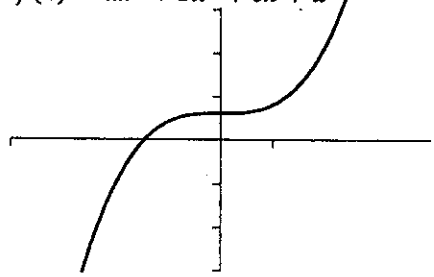
$f(x) = ax^2 + bx + c$



Domain: $(-\infty, \infty)$ **Range:** $(0, \infty)$
x-intercept: $\left(\frac{-b \pm \sqrt{b^2-4ac}}{2a}, 0\right)$ **y-intercept:** $(0, c)$

Cubic Function

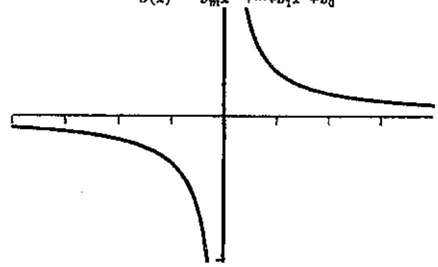
$f(x) = ax^3 + bx^2 + cx + d$



Domain: $(-\infty, \infty)$ **Range:** $(-\infty, \infty)$
y-intercept: $(0, d)$

Rational Function

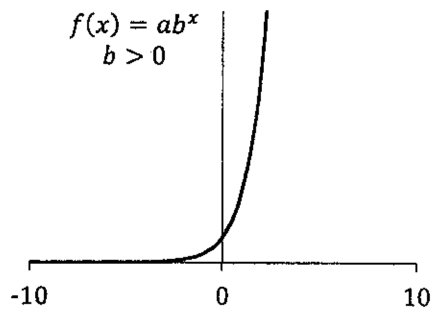
$f(x) = \frac{M(x)}{D(x)} = \frac{a_n x^n + \cdots + a_1 x^1 + a_0}{b_m x^m + \cdots + b_1 x^1 + b_0}$



Domain: all real numbers, $D(x) \neq 0$ **Range:** $(-\infty, 0), (0, \infty)$
x-intercept: zeros of $N(x)$ **y-intercept:** $(0, f(x))$ if $f(0)$ exists
Vertical Asymptote: zeros of $D(x)$
Horizontal Asymptote: $y = 0$ if $n < m$
$y = a_n/b_m$ if $n = m$

Exponential Increasing Function

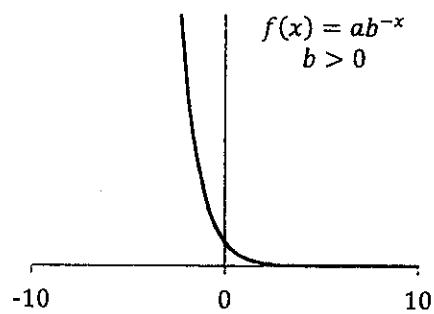
$f(x) = ab^x$
$b > 0$



Domain: $(-\infty, \infty)$ **Range:** $(0, \infty)$
x-intercept: none **y-intercept:** $(0, a)$
Horizontal Asymptote:

Exponential Decreasing Function

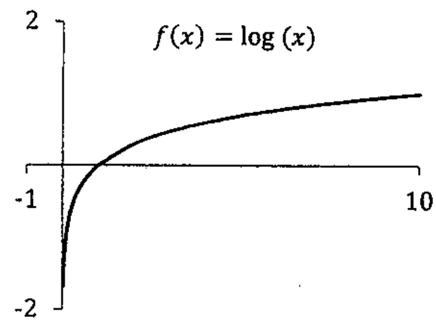
$f(x) = ab^{-x}$
$b > 0$



Domain: $(-\infty, \infty)$ **Range:** $(0, \infty)$
x-intercept: none **y-intercept:** $(0, a)$
Horizontal Asymptote: $y = 0$

Logarithmic Function

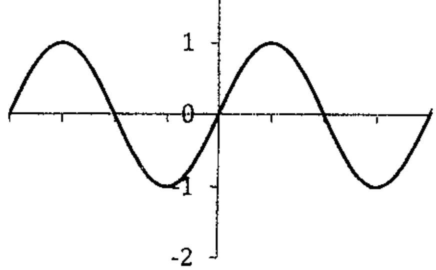
$f(x) = \log(x)$



Domain: $(0, \infty)$ **Range:** $(-\infty, \infty)$
x-intercept: $(1,0)$ **y-intercept:** none
Vertical Asymptote: $x = 0$

Sine Function

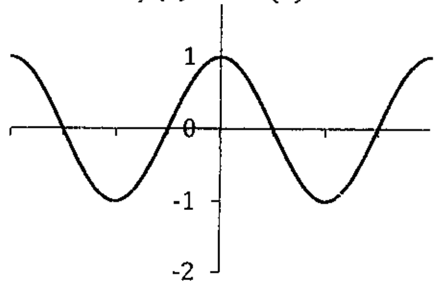
$f(x) = \sin(x)$



Domain: $(-\infty, \infty)$ **Range:** $[-1,1]$
Period: 2π

Cosine Function

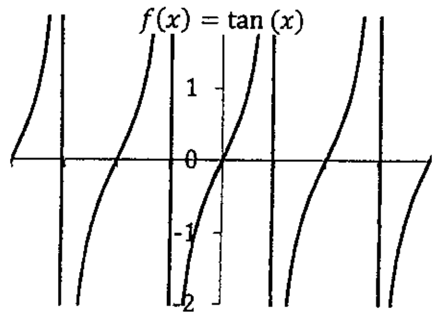
$f(x) = \cos(x)$



Domain: $(-\infty, \infty)$ **Range:** $(-1,1)$
Period: 2π

Tangent Function

$f(x) = \tan(x)$



Domain: all real numbers except $x \neq \pi/2 + n\pi$ **Range:** $(-\infty, \infty)$
Vertical Asymptotes: $x = \pi/2 + n\pi$
Period: π