Solubility of Salts - $K_{sp}$

We now focus on another aqueous equilibrium system, slightly soluble salts. These salts have a Solubility Product Constant, $K_{sp}$.
(We saw this in 1B with the sodium tetraborate solubility lab.)

For example, $K_{sp}$ is defined for Cu(OH)$_2$(s) as follows:

$$
\text{Cu(OH)}_2(\text{s}) \rightleftharpoons \text{Cu}^{2+}(\text{aq}) + 2 \text{OH}^- (\text{aq})
$$

$$
K_{sp} = [\text{Cu}^{2+}] [\text{OH}^-]^2 = 4.8 \times 10^{-20} \text{ at 25 °C}
$$

For a list of $K_{sp}$ values at 25°C, refer to Appendix D-Table D.3.

These slightly soluble salts dissolve until $K_{sp}$ is satisfied. At this point, we say the solution is saturated, and no more salt will dissolve. In terms of $Q_{sp}$ we have the following possibilities:

1. $Q_{sp} < K_{sp}$
2. $Q_{sp} = K_{sp}$
3. $Q_{sp} > K_{sp}$

K$_{sp}$ Solubility

Write the chemical equilibrium and the $K_{sp}$ equilibrium-constant expression for the solubility of Ca$_3$(PO$_4$)$_2$.

Solubility and $K_{sp}$

- The solubility of salts (saturated solution) are often expressed in one or more of the following units with the temperature also specified:
  - mol/L (molar solubility)
  - g/L, g/mL, g/100 mL or mg/L (Note that these are NOT densities.)
  - ppm or ppb (parts per million or parts per billion) for very insoluble salts.
- The above units for solubility are directly related to $K_{sp}$ through equilibrium stoichiometry. Let’s do some examples.
Solubility and $K_{sp}$

1. What is the molar solubility of $\text{Ca}_3(\text{PO}_4)_2$ at 25°C given a $K_{sp}$ of $2.0 \times 10^{-29}$?

1.1. What is the concentration of calcium ions?

1.2. What is this solubility in g/L solution?

1.3. What is this solubility in parts per million (ppm) and parts per billion (ppb)? (Assume a soln density of 1.0 g/mL)

$$ppm = \frac{\text{mass of component in soln}}{\text{total mass of soln}} \times 10^6$$

$$ppb = \frac{\text{mass of component in soln}}{\text{total mass of soln}} \times 10^9$$

$K_{sp}$ from Experimental Data

A saturated solution of magnesium hydroxide in water has a pH of 10.38. Calculate the $K_{sp}$ for magnesium hydroxide.
Relative Molar Solubility of Salts

The relative MOLAR solubility of salts (saturated solution) can be determined by comparing \( K_{sp} \) values. The greater the \( K_{sp} \) the more ions are in solution, hence the greater the molar solubility.

However, you can only directly compare salts that give equivalent numbers of ions in solution.

For example, you can compare the \( K_{sp} \)'s of all salts with a 1:1 ion ratio like: AgBr, BaSO\(_4\) etc.

Or, you could compare \( K_{sp} \)'s of all salts with 1:2 and 2:1 ion ratios like: BaCl\(_2\), Ag\(_2\)SO\(_4\), etc.

But you can't compare the \( K_{sp} \) AgBr directly to the \( K_{sp} \) Ag\(_2\)SO\(_4\) to determine which is more soluble.

Which salt has a greater molar solubility soluble: AgBr or BaSO\(_4\)?

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K\(_sp\) Values and Solubility Calcs. LIMITATIONS EXIST!

Unfortunately, solubilities calculated using \( K_{sp} \) values sometimes deviate appreciably from the experimentally measured solubilities! Why?

**Reason 1 of 2:** Some salts do not completely dissociate 100% into their respective ions. For example, PbCl\(_2\) exists in three different forms in water, each with its own individual \( K \) value.

1. PbCl\(_2\)(s) ⇌ PbCl\(_2\)(aq) (no ions) \hspace{1cm} K = 1.1\times10^{-3}; \hspace{0.5cm} S = 0.0011
2. PbCl\(_2\)(s) ⇌ Pb\(^{+}\)(aq) + Cl\(^-\)(aq) (two ions) \hspace{1cm} K = 6.9\times10^{-4}; \hspace{0.5cm} S = 0.026
3. PbCl\(_2\)(s) ⇌ Pb\(^{2+}\)(aq) + 2 Cl\(^-\)(aq) (all ions) \hspace{1cm} K = 1.7\times10^{-5}; \hspace{0.5cm} S = 0.016

The experimental solubility of PbCl\(_2\) is 0.036 M. Over twice the value predicted by \( K_{sp} \) alone.
Reason 2: Some anions in salts are strongly basic. They react with water in a \( K_b \) equilibrium. This reduces the anion concentration available to satisfy \( K_{sp} \). This in turn increases the solubility of the salt.

The three common ions that are basic enough to have considerable reaction with water are: \( S^{2-} \), \( CO_3^{2-} \), and \( PO_4^{3-} \).

For example consider \( Ca_3(PO_4)_2 \):
The \( K_{sp} \) equilibrium is:

\[
Ca_3(PO_4)_2 (s) \rightleftharpoons 3 \text{Ca}^{2+}(aq) + 2 \text{PO}_4^{3-}(aq) \quad \text{\( K_{sp} = 2.0 \times 10^{-29} \)}
\]

Write the chemical reaction for the \( K_b \) equilibrium (hydrolysis) for \( \text{PO}_4^{3-}(aq) \):

\[ K_b = ? \] (How do you determine the value?)

The experimental solubility of \( Ca_3(PO_4)_2 \) is somewhat higher than predicted by \( K_{sp} \) alone because some phosphate ion is removed from solution through the \( K_b \) reaction.

\( K_{sp} \) values are still useful for estimating solubilities, predicting trends and predicting relative solubilities. We just need to keep in mind that they have limitations!

We previously determined the molar solubility of \( Ca_3(PO_4)_2(s) \) using \( K_{sp} \) alone. Now let’s determine its solubility including \( K_b \) with \( K_{sp} \) to find \( K_{net} \).

The \( K_{sp} \) of Sulfide Salts INCLUDES \( K_b \)

Since the sulfide ion is very basic, \( K_b \) significantly increases the solubility of sulfide salts. For the metal sulfides, the \( K_{sp} \) values are actually \( K_{net} \) for the overall equilibrium that includes two processes:

1) the dissolution of the metal sulfide

2) the \( K_b \) hydrolysis reaction of the the sulfide ion.

(See the footnote at the bottom of Table D.3 in our text book)

For CuS write the chemical reaction for each of the above equilibria, and for the net equilibrium:

1. \( K_1 \)
2. \( K_b >> 1 \)
Net: \( K_{sp} = K_1 \cdot K_b = 6 \times 10^{-37} \) (Table D.3)

Determine the pH of a saturated CuS solution.
• Like the percent ionization of weak acids and bases, the solubility of salts is influenced by the presence of a common ion. These salts are less soluble when a common ion is present just like a weak acid’s ionization is limited in the presence of significant conjugate base.

1. The $K_{sp}$ of silver iodide in water is $8.3 \times 10^{-17}$ M. Calculate the molar solubility of silver iodide in:
   1. Pure water
   2. 0.010 M NaI (Common Ion Effect)

Solubility of Salts and the **Common Ion Effect**

1. The barium ion, $\text{Ba}^{2+}(aq)$, is poisonous when ingested. The lethal dose in mice is about 12 mg $\text{Ba}^{2+}$ per kg of body mass. Despite this fact, $\text{BaSO}_4$ is widely used in medicine to obtain X-ray images of the gastrointestinal tract since Ba is a very good x-ray absorbing element.
   a) Explain why $\text{BaSO}_4(s)$ is safe to ingest, even though $\text{Ba}^{2+}(aq)$ is poisonous.
   
   b) Calculate the concentration of $\text{Ba}^{2+}$, in milligrams per liter, in saturated $\text{BaSO}_4(aq)$ at 25°C.
   
   c) Soluble $\text{MgSO}_4$ is often mixed with $\text{BaSO}_4$ when ingested. What function does the $\text{MgSO}_4$ serve?
Solubility of Salts with *pH* Adjustments

We have already seen that basic salts containing $S^{2-}$, $CO_3^{2-}$, and $PO_4^{3-}$ ions are more soluble in water than expected because of the reaction of the basic anion with water through $K_b$.

- All salts that contain a basic anion will have their solubility increased in an acidic solution. In acidic solutions the basic anion reacts with the acid, forcing more of the salt to dissolve to reach equilibrium.

For example, PbF$_2$ and Mg(OH)$_2$ are practically insoluble in water. However, they are very soluble in dilute acids.

1. Which of the following slightly soluble salts will be more soluble in acidic solution than pure water? For those where the solubility increases, write the net ionic chemical reaction that occurs when a strong acid is present in solution:

   a) $Al(OH)_3(s)$

   b) $BaSO_4(s)$

   c) $BaC_2O_4(s)$

   d) $PbCl_2$
Solubility and \( pH \)

1. Calculate the molar solubility of iron (II) hydroxide \((K_{sp} = 7.9 \times 10^{-16})\) in
   a) A solution buffered at \( pH = 7.00 \):
   
   b) What could you say about the solubility of iron (II) hydroxide in a solution buffered at \( pH = 10.00 \)?

Solubility and \( pH \)

1. A chief component in marble is calcium carbonate. Marble has been widely used for statues and ornamental work on buildings, but marble is readily attacked by acids. Assume that the overall reaction that occurs in a dilute acid is

\[
\text{CaCO}_3(s) + \text{H}_3\text{O}^+(aq) \rightleftharpoons \text{Ca}^{2+}(aq) + \text{HCO}_3^-(aq) + \text{H}_2\text{O}(l)
\]

Determine the equilibrium constant for this reaction* and then determine the solubility of calcium carbonate in a buffer with \( pH = 5.6 \) (\( pH \) stays constant).

*Hint: What combination of reactions yields the overall reaction?
Solubility of Salts and pH-Applications

A principle cause of sinkhole formation: dissolution of limestone (CaCO$_3$) by naturally acidic rain water as it percolates through the bedrock. Voids in the bedrock can cause sudden collapse of the overlying ground.

Sinkhole Formation

The large sinkhole shown here has destroyed several buildings and part of a highway.
1. Two of the main crystalline components of kidney stones are calcium phosphate and calcium oxalate. What happens to the solubility of these compounds as pH is increased?

2. Tooth enamel consists mainly of a mineral called hydroxyapatite, Ca$_5$(PO$_4$)$_3$(OH)(s), $K_{sp} = 6.8 \times 10^{-37}$, that is insoluble in pure water. When acids dissolve tooth enamel, the result is tooth decay. Write the net ionic equation that occurs between hydroxyapatite and H$_3$O$^+$(aq). Note, the phosphate ions are also pronated along with the hydroxide.

3. Fluoridation of water and the use of fluoride toothpaste causes the OH$^-$ ion in hydroxyapatite to be replaced with F$^-$ forming Ca$_5$(PO$_4$)$_3$F ($K_{sp} = 6.8 \times 10^{-60}$). Suggest a reason why fluoride helps prevent tooth decay.

Complex Ion Formation, $K_f$

Complex ions are ions that usually contain a transition metal cation and one or more ligands. A ligand is either a neutral molecule or an anion that bonds to the metal cation through a Lewis acid-base reaction:

- Metal Ion: Lewis acid - LIMITING REACTANT
- Ligand: Lewis Base - EXCESS REACTANT

\[
\text{Ag}^+(aq) + 2 \text{NH}_3(aq) \rightleftharpoons \text{Ag(NH}_3)_2^+(aq) \quad K_f = 1.7 \times 10^7 = ? \text{ (Write the expression)}
\]

- What can you conclude about the favorability of complex ion formation?
- What can you conclude about the free (uncomplexed) metal cation concentration in solutions where a complex ion can form and the ligand is in excess?
**Complex Ion Formation, \( K_f \)**

Determine the concentration of free (uncomplexed) silver ions in solution when 5.0 mL of a 6.0 M ammonia solution (ligand) is added to 45.0 mL of a 0.10 M silver nitrate solution (metal cation).

---

**Complex Ion Formation and Solubility**

Formation of complex ions is one way to dissolve “insoluble” salts. Consider the following sequence of additions to a solution containing \( \text{Ag}^+ \) ions.

- **Precipitation of AgCl**
  - AgCl is precipitated by adding NaCl(aq) to AgNO\(_3\)(aq)

- **Complex ion formation**
  - \( \text{AgCl}(s) \rightarrow [\text{Ag}(\text{NH}_3)\text{Cl}]^+ (aq) \)

- **Precipitation of AgBr**
  - The AgCl is dissolved by adding excess aqueous NH\(_3\)
  - The silver-ammonia complex ion is changed to insoluble AgBr on addition of NaBr(aq).

- **Complex ion formation**
  - AgBr precipitated by adding excess Na\(_2\)S\(_2\)O\(_3\)(aq)
  - The AgBr is dissolved by adding excess Na\(_2\)S\(_2\)O\(_3\)(aq).
Complex Ion Formation and Solubility

Using equilibrium constants ($K_{sp}$ and $K_f$) we can show why each process in the previous illustration can be made favorable.

**Step (a):** Precipitation of AgCl ($K_{sp} = 1.8 \times 10^{-10}$)

\[
\text{Ag}^+(aq) + \text{Cl}^-(aq) \rightleftharpoons \text{AgCl(s)} \quad K_{\text{ppt}} = ?
\]

What concentration of Cl$^-$ (aq) is required to precipitate 99.9% of the Ag$^+$ (aq) ions from a 0.10 M AgNO$_3$(aq) solution. What mass of NaCl (molar mass = 58.44 g/mol) must be added to 100.0 mL of the solution to accomplish this? (Assume that the volume change of the solution is negligible upon addition of the NaCl.)

**Step (b):** AgCl(s) dissolves when NH$_3$(aq) is added:

\[
\text{AgCl(s)} \rightleftharpoons \text{Ag}^+(aq) + \text{Cl}^-(aq) \quad K_{sp} = 1.8 \times 10^{-10}
\]

\[
\text{Ag}^+(aq) + 2 \text{NH}_3(aq) \rightleftharpoons \text{Ag(NH}_3)_2^+(aq) \quad K_f = 1.7 \times 10^7
\]

\[
\text{AgCl(s)} + 2 \text{NH}_3(aq) \rightleftharpoons \text{Ag(NH}_3)_2^+(aq) + \text{Cl}^-(aq) \quad K_{\text{net}} = ?
\]

A high concentration of the ligand, NH$_3$(aq), in solution ensures that almost all the AgCl(s) will dissolve. This is an application of Le Chatelier’s Principle.

This is a key point for dissolving insoluble salts by formation of a complex ion:

Salt + Ligand $\rightarrow$ Complex ion + Anion $\quad K_{\text{net}} = K_{sp} \times K_f$
**Calculations: Complex Ion Formation and Solubility**

**Step (b):** What is the molar solubility of AgCl(s) in
a) pure H₂O(l):

b) a solution where the concentration of NH₃(aq) begins at 0.10 M:

---

**Calculations: Complex Ion Formation and Solubility**

**Step (c):** Ag(NH₃)₂⁺(aq) precipitates as AgBr(s) when NaBr(aq) is added.

Find $K_{net}$ for this system.

Determine the concentration of complexed ion still in solution where the Ag(NH₃)₂⁺(aq) and NaBr(aq) concentrations both begin at 0.10 M.
Complex Ion Formation and Solubility

**Step (d):** AgBr(s) dissolves as Na$_2$S$_2$O$_3$(aq) is added. (Write the net ionic equation for the reaction below.)

Crystals of AgBr can be removed from black and white photographic film by reacting the AgBr(s) with aqueous sodium thiosulfate. In order to dissolve 2.5 g of AgBr in 1.00 L of solution, what concentration of thiosulfate ion is needed in solution at equilibrium?

Now calculate how many grams of solid Na$_2$S$_2$O$_3$ (molar mass = 158.11g/mol) must be added to dissolve the AgBr.

Amphotericism of Metal Oxides and Hydroxides

Definition of amphoteric: An amphoteric substance is slightly soluble in water, but soluble in either acidic or basic solutions.

Example Al$^{3+}$: (waters of hydration omitted for clarity)

1. $\text{Al(OH)}_3(s) + 3 \text{H}^+(aq) \rightleftharpoons \text{Al}^{3+}(aq) + 3 \text{H}_2\text{O}(l)$ Acidic solution, pH < 7
2. $\text{Al}^{3+}(aq) + \text{OH}^-(aq) \rightleftharpoons \text{AlO}^2-(aq)$ Weakly basic solution, 7 < pH < 9
3. $\text{AlO}^2-(aq) + \text{OH}^-(aq) \rightleftharpoons \text{Al(OH)}_2^+(aq)$ Basic solution, pH ≈ 10
4. $\text{Al(OH)}_2^+(aq) + \text{OH}^-(aq) \rightleftharpoons \text{Al(OH)}_3(s)$ Strongly basic solution, pH > 10
5. $\text{Al(OH)}_3(s) + \text{OH}^-(aq) \rightleftharpoons \text{Al(OH)}_4^-(aq)$

Other amphoterics:
- Cr$^{3+}$, Zn$^{2+}$, and Sn$^{2+}$
- Each will complex a different # of hydroxide ions - you will have to check a table of $K_f$ values.

Other Aspects of Ionic Equilibria
Zinc hydroxide is insoluble in water \((K_{sp} = 3.0 \times 10^{-16})\), but amphoteric. Write net ionic equations to show why Zn(OH)\(_2\) is readily soluble in the following solutions:

(a) \(\text{CH}_3\text{COOH(aq)}\)

(b) \(\text{NH}_3\text{(aq)}\): Zn\(^{2+}\) ions form the following complex ion: \([\text{Zn(NH}_3)_4]^{2+}\) with a \(K_f = 4.1 \times 10^8\)

(c) \(\text{OH}^-(aq)\) Zn\(^{2+}\) ions form the following complex ion: \([\text{Zn(OH)}_4]^{2-}\) with a \(K_f = 4.6 \times 10^{17}\)

(c.1) What is the minimum pH required to dissolve 1.5 g of Zn(OH)\(_2\) in a liter of solution?

---

Recall from previous slides about solubility:

\(Q_{sp} < K_{sp}\) : unsaturated soln. - no precipitation.

\(Q_{sp} = K_{sp}\) : saturated soln. - equilibrium.

\(Q_{sp} > K_{sp}\) : supersaturated soln. - a ppt. should form.

Using \(K_{sp}\) values:

1. We can determine if a ppt will form when two solutions are mixed that contain a cation and an anion that can combine to form a slightly soluble salt.

2. We can determine if different cations in solution can be quantitatively separated from each other by selective precipitation. In other words, we can determine if one cation can be removed from the solution as a precipitated salt before a second cation in solution is also precipitated.
calcium ion triggers clotting of blood. Therefore, when blood is donated the receiving bag contains sodium oxalate to precipitate the Ca$^{2+}$, and thus prevent clotting. Typically, blood contains $9.7 \times 10^{-5}$ g Ca$^{2+}$/mL. To remove the calcium ions, a medical technologist treats a 104-mL blood sample with 100.0 mL of 0.155 M Na$_2$C$_2$O$_4$. Calculate the [Ca$^{2+}$] left in solution after the treatment. ($K_{sp}$ for calcium oxalate = 2.6x10$^{-9}$)

**Single Precipitation Reactions**

1. Show that a precipitate of Mg(OH)$_2$ will form in an aqueous solution that is 0.350 M MgCl$_2$ and 0.750 M NaOH.

2. Show that a precipitate of Mg(OH)$_2$ will also form in an aqueous solution that is 0.350 M MgCl$_2$ and 0.750 M NH$_3$. Hint: Find the [OH$^-\)] produced by the hydrolysis of NH$_3$. Is it enough to ppt the Mg$^{2+}$ ion?

2.1. Explain why the Mg(OH)$_2$ precipitate can be prevented from forming in (2) by adding NH$_4$Cl to the solution containing ammonia.
Single Precipitation Reactions

Calculate the minimum pH needed to precipitate Mn(OH)₂ so completely that the concentration of Mn²⁺ is less than 1 µg per liter, that is 1 part per billion (ppb).

Selective Precipitation of Cations

Differences in molar solubilities between compounds containing a common ion can be used to selectively precipitate one ion from solution leaving the other ion in solution. We can calculate the amount of the common ion needed to reach saturation for the most soluble ionic salt. Addition of just a little less of the common ion will insure the most complete separation possible.

Quantitative, or complete, separation is considered possible if 99.9% of the least soluble salt precipitates before any of the most soluble salt starts to precipitate. Typically, the added reagent (containing the common ion) is quite concentrated so that its addition does not appreciable change the volume of the solution and dilute the solution containing the cations to be separated.

Solution of cations $\xrightarrow{\text{Add ppt agent}}$ Least soluble cation ppt $\xrightarrow{\text{Centrifuge and decant}}$ Selective ppt complete

A 400.0 mL solution has $[\text{Ba}^{2+}] = 0.0040 \text{ M}$ and $[\text{Sr}^{2+}] = 0.0010 \text{ M}$. A 0.800 M Na₂SO₄(aq) solution is added drop-wise.

Which ion, Ba²⁺ or Sr²⁺, will precipitate first? $K_{sp}$ for BaSO₄ is $1.1 \times 10^{-10}$, $K_{sp}$ for SrSO₄ is $3.4 \times 10^{-7}$.

Can the two cations be quantitatively (99.9%) separated by selective precipitation?
Applications

Precipitation Reactions:
- Precipitation of Mg\(^{2+}\) as Mg(OH)\(_2\) from sea water is the source of Mg metal.
- Titration of solutions containing Cl\(^{-}\) with AgNO\(_3\) to quantitatively determine [Cl\(^{-}\)]. For this to work, we must be sure that almost all of the Cl\(^{-}\) precipitates as we add the AgNO\(_3\). How can we be sure of this?
- Determination of the amount of SO\(_4^{2-}\) in solution by precipitating as BaSO\(_4\). The BaSO\(_4\) formed is filtered off, dried, and weighed. For this to work, the precipitation of SO\(_4^{2-}\) must be complete (99.9%). How can we be sure of this?

Complex Ion Formation:
- Complex ion formation can be used to extract gold from low-grade gold containing rock. The formation constant of Au(CN)\(_2^{-}\) is very large. A very small concentration of Au\(^{+}\) ions are formed through oxidation of Au. This oxidation takes place in the presence of CN\(^{-}\) ions. Complex ion formation removes the Au\(^{+}\) ions from solution, so more are formed. As a result, even though Au is not oxidized normally by air, bubbling air through a suspension of Au containing ore in the presence of CN\(^{-}\) leads to formation of a solution of the complex ion:
  \[
  4\text{Au(s)} + 8\text{CN}^{-}\text{(aq)} + \text{O}_2\text{(g)} + 2\text{H}_2\text{O(l)} \rightarrow 4\text{Au(CN)}_2^{-}\text{(aq)} + 4\text{OH}^{-}\text{(aq)}
  \]
  The resulting solution is filtered and the Au\(^{+}\) reduced to Au(s).
- Aluminum ore contains Fe\(_2\)O\(_3\) impurities along with the desired aluminum hydroxide, Al\(_2\)O\(_3\). A strong base is added to dissolve the Al\(_2\)O\(_3\) as the Al(OH)\(_4^{-}\) complex ion (K\(_f\) = 3x10\(^{-33}\)). The iron (III) ion does not form a complex ion with hydroxide, therefore the Fe\(_2\)O\(_3\) does not dissolve. The resulting solution is filtered and acid is added to the filtrate to precipitate Al(OH)\(_3\). The filtrate must not be made too acidic or the Al(OH)\(_3\) will redissolve!