Introduction

In this tutorial you will learn to use conversion factors to solve a variety of problems that arise in chemistry. Roughly 90% of all the mathematics done in beginning chemistry involves using one or more conversion factors. Conversion factors come in a many forms. For the novice student recognizing and extracting conversion factors from information given in a problem is often the most difficult step. As you follow along with the examples presented keep in mind what other conversion factors you might be able to generate from the information given.

Conversion Factors and Equivalencies

A conversion factor, in essence, is any ratio that relates one quantity or amount to another quantity or amount. An example is speed, say 25 mi/hr. Distance (25 mi) is related to time (1 hr) as a ratio. This conversion factor can be used to

1. predict the distance traveled given 2.2 hr of time:
   $$2.2\cancel{hr}\left(\frac{25mi}{1\cancel{hr}}\right) = 55mi$$
2. predict the time of travel given a distance of 94 mi:
   $$94\cancel{mi}\left(\frac{1hr}{25\cancel{mi}}\right) = 3.8hr$$ (notice the inverse of the speed was used)

An equivalency is a ratio of terms – equivalent to the number 1. Equivalencies are a special type of conversion factor – they are used to change from one unit into another **of the same type** (e.g. length to length, mass to mass, volume to volume, etc). For example, when converting miles (mi) to kilometers (km) (or vice-versa) we need to use an equivalency. You might find one of the following equivalencies in your textbook:

1 km = 0.6214 mi   or   1.609 km = 1 mi

Either of these equivalencies can be used to convert between mi and km, in fact they are inverses of each other. To show that equivalencies represent equal values and have a ratio equal to 1, divide both sides of the first equivalency by 1 km:

$\frac{1\cancel{km}}{1\cancel{km}} = \frac{0.6214mi}{1km} = 1$. So 0.6214 mi must be the same distance as 1 km since their ratio is 1.

To use any equivalency (or conversion factor) make sure your units cancel as you read left to right. To convert 16.2 mi to km I must write the equivalency with km as the numerator (top) and mi as the denominator (bottom):

$$16.2\cancel{mi}\frac{1km}{0.6214\cancel{mi}} = 26.1km$$

In chemistry, the most important equivalency is the mole (mol, n). This is the chemist's counting unit, just like a baker uses a dozen to count, 1 dozen = 12, chemists count atoms and molecules moles at a time. The mole is a really big number:

1 mole = 6.0221×10^{23}

Notice the mole has no units, it represents the number of items you are counting. You can count anything you like, not just atoms and molecules. As an example, it is estimated that the universe contains 70 sextillion or some 70 thousand-million-million-million visible stars. That's a 7 followed by 22 zeros. How many moles of visible stars is this?

$$7x10^{22}\,\cancel{stars}\left(\frac{1 mole\ stars}{6.0221x10^{23}\,\cancel{stars}}\right) = 0.1 mole\ stars$$. Heck, only one tenth of a mole of stars,

not very many! Compare that to a 1 liter bottle of water that contains ≈55 moles of water molecules! On a relative scale there are 550 times more water molecules in a 1 liter bottle than all the visible stars in the universe! Imagine the number of molecules of water in all the oceans!

A note of caution!

Before we begin using conversion factors I must stress the following:

YOU CANNOT BE SUCCESSFUL WITH UNIT CONVERSIONS UNLESS YOU ARE WILLING TO ACTUALLY WRITE THE UNITS IN EVERY STEP!

After years of teaching I see students fall apart when solving a problem because they simply refuse to write down any units. Numbers alone without units have no context or meaning. In my opinion, the units are more important then the actual numbers. Why? The units themselves will lead you to a correct sequence of conversions to solve the problem. Once the unit analysis is done and the conversion factors are in place, any input number can be used to generate an answer. Unit analysis is the key to using conversion factors properly!

How to solve a problem.

Here are a few pointers on how to start and then solve a problem.

1. **Read the problem carefully!** This means understanding what is given in the problem and more importantly what is asked for in the answer.
    a. Rewrite all the given information (with units) on the margin of the paper.
    b. Look at what is asked, what are the units?
    c. Are any equivalencies given, any conversion factors?
2. **Make a plan**, in detail on how you are going to solve the problem. Most of the steps in this plan will involve CONVERSION FACTORS! Have a visual map of the steps necessary to generate the final answer. Write it down, it is very helpful! Think about the following as you plan:
    a. Is there any information missing?
    b. What conversion factors will I need?
    c. Will I have to generate my own conversion factors?
    d. As you develop your plan, **write the units down!** This will keep you on track and alert you when your plan is incorrect or lacking in detail.
3. Solve the problem. **Check that your answer is reasonable!** If you are asked to convert ft^3 to cm^3, you should have reasoned ahead of time the following:
    a. The centimeter is smaller than the foot. I will have more cm^3 than ft^3.
    b. My answer cannot be negative.
4. If you cannot solve a problem **get help immediately!** Believe me, the problems will not get easier, any stumbling block you have early on will be magnified later.

Using conversion factors & equivalencies– the unit canceling method

When making your plan to solve a problem rely on the units to guide you to your next step. Using the units canceling method will surely help you find the next step in the plan. Let's first start with simple equivalencies and then move to more complicated conversion factors. The methodology to solve a problem varies, so I will teach by example, not a set of rules. I will follow the pointers given above.

Equivalencies

As sated, we will use equivalencies when converting within the same type of unit, time for example.

A. Convert 1 365 day year to seconds. When done complete the equivalency 1 yr = ? s.

1. Did you read the problem carefully?
   - **a)** The starting unit is years.
   - **b)** The output unit is seconds.
2. Make a plan. I made the following plan based on units only:
   yr → day → hr → min → s
   - **a)** To solve the problem I will need four equivalencies, one for each step. Look them up if you don't know them. Here are the four I'm using:
     365 days = 1 year; 24 hours = 1 day; 60 minutes = 1 hour; 60 seconds = 1 min.
3. **Unit cancellation makes my conversion map, no numbers yet**. I map my plan with units only, making sure units cancel as I proceed left to right. The units in the numerator on the left will appear in the denominator of the conversion factor to the right:

$$? s = \cancel{yr}\left(\frac{\cancel{day}}{\cancel{yr}}\right)\left(\frac{\cancel{hr}}{\cancel{day}}\right)\left(\frac{\cancel{min}}{\cancel{hr}}\right)\left(\frac{s}{\cancel{min}}\right)$$

   - **a)** Add the numbers to the conversion factors from the equivalency statements:

Starting unit | Convert yr to day | Convert day to hr | Convert hr to min | Convert min to s | Final answer

$$? s = \cancel{yr}\left(\frac{365\,\cancel{day}}{1\,\cancel{yr}}\right)\left(\frac{24\,\cancel{hr}}{1\,\cancel{day}}\right)\left(\frac{60\,\cancel{min}}{1\,\cancel{hr}}\right)\left(\frac{60\,s}{1\,\cancel{min}}\right) = 31536000 \text{ s.}$$

   - **b)** You now have a new equivalency statement:
     1 yr = 31536000 s.
4. Check, does it make sense? Yes, millions of seconds in a year!

B. Now for another example. There are 32 fluid ounces (fl oz) in a US quart (qt). Express the volume of a 750-cm^3 bottle in liters, ounces and quarts.

1. Did you read the problem carefully?
   - **a)** The starting unit is cm^3.
   - **b)** The output units are liters (L), ounces and quarts.
2. Since three different units are required intermediate answers will be used as inputs to the next conversion. So the plan is as follows:
   cm^3 → L → qt → fl oz

   a) To solve the problem I need three equivalencies: (Look them up!)
   $1000\ cm^3 = 1$ L; 0.9461 L = 1 qt; 32 fl oz = 1 qt.

3. **Keep intermediate results.** The result of each conversion is used in the next step;
   a) $750\text{-}cm^3\left(\dfrac{1L}{1000cm^3}\right) = 0.750$ L **b)** $0.750\ L\left(\dfrac{1qt}{0.9461L}\right) = 0.79\underline{2}8$ qt
   c) $0.79\underline{2}8\ qt\left(\dfrac{32\,fl\ oz}{1qt}\right) = 25.4$ fl oz
4. Check, does it make sense? Yes since a liter and quart are very close in volume.

C. Now for a twist. Many times you will be asked to convert between units like area or volume that are not "linear" units, they are squared (area) or cubed (volume) length units. ***The trick to making these conversions is to set them up as though they were linear conversions and then convert them to the correct (nonlinear) units***.
As an example I'll construct an equivalency to convert 25.6 cubic mm (mm^3) to cubic yards (yd^3), that is, $?yd^3 = 25.6\ mm^3$.

1. Did you read the problem carefully?
   a) The starting unit is mm^3.
   b) The ending unit is yd^3
   c) These are nonlinear units! Extra work will be required!
2. The plan will be based on converting the linear length unit mm to yd. **I will ignore the cubic power for now**:
   mm → cm → in → ft → yd
   a) Gather the necessary **LINEAR equivalencies**:
   10 mm = 1 cm; 2.54 cm = 1 in; 12 in = 1 ft; 3 ft = 1 yd
      i) Construct a LINEAR conversion sequence of units, B**O NEED to ADD the 25.6 yet.**
      $$?\,yd = mm\left(\frac{1cm}{10mm}\right)\left(\frac{1in}{2.54cm}\right)\left(\frac{1ft}{12in}\right)\left(\frac{1yd}{3ft}\right)$$
      ii) Now CUBE everything in this linear unit conversion construction. **Carry the cube power to yd and mm and each equivalency:**
      $$?\,yd^3 = mm^3\left(\frac{1cm}{10mm}\right)^3\left(\frac{1in}{2.54cm}\right)^3\left(\frac{1ft}{12in}\right)^3\left(\frac{1yd}{3ft}\right)^3$$
      iii) Move the cubic power into the parentheses of each equivalency, **making sure BOTH NUMBERS and UNITS are cubed:**
      $$?\,yd^3 = mm^3\left(\frac{(1cm)^3}{(10mm)^3}\right)\left(\frac{(1in)^3}{(2.54cm)^3}\right)\left(\frac{(1ft)^3}{(12in)^3}\right)\left(\frac{(1yd)^3}{(3ft)^3}\right)$$
      iv) Now, **apply the cube to every number (except 1's) and write the units as cubed units**:
      $$?\,yd^3 = mm^3\left(\frac{1cm^3}{10^3mm^3}\right)\left(\frac{1in^3}{2.54^3cm^3}\right)\left(\frac{1ft^3}{12^3in^3}\right)\left(\frac{1yd^3}{3^3ft^3}\right)$$
      v) Attach the value to be converted (25.6) to the leading mm^3 units:
      $$?\,yd^3 = 25.6mm^3\left(\frac{1cm^3}{10^3mm^3}\right)\left(\frac{1in^3}{2.54^3cm^3}\right)\left(\frac{1ft^3}{12^3in^3}\right)\left(\frac{1yd^3}{3^3ft^3}\right)$$

3. **3.** Now carry out the math with a calculator as written above or rewrite the equivalencies with the result of each number being cubed (as shown below):

$$?\,yd^3 = 25.6mm^3\left(\frac{1cm^3}{1000mm^3}\right)\left(\frac{1in^3}{16.39cm^3}\right)\left(\frac{1ft^3}{1728in^3}\right)\left(\frac{1yd^3}{27ft^3}\right)$$

a) Completing the math:
$3.35x10^{-8}\,yd^3 = 25.6mm^3$

4. Check, does it make sense? Yes since a yd is much larger than a mm, there will be much, much less than one yd^3 in 25.6 mm^3.

5. As a final note, the method above will work with any power, not just squares and cubes.

Conversion factors – units of different types

Now lets consider the more general case of conversion factors that convert between units of a different type. These are very common in the sciences and often hard to recognize in the context of a problem. Some examples of these types of conversion factors in chemistry are:

1. **Density**: ($d=g/cm^3$) converts between volume and mass.
2. **Molar mass**: (M=g/mol) Converts between moles (amount) and grams (mass).
3. **Molarity** (M=mole/L): converts between volume (L) and moles of solute.
4. **Mass %X in Y**: converts between the mass of a part, X, and the mass of the whole, Y.
5. **Stoichiometric coefficients**: **2** Al + **3** Cl_2 → **2** $AlCl_3$ convert between moles of one reactant/product to moles of another reactant/product.
6. **Subscripts** in a chemical formula: $C_{\mathbf{6}}H_{\mathbf{6}}O$, convert between moles of one element to moles of another element or moles of the formula unit.

D. An example from chemistry that uses many of the conversion factors listed above might read as follows: A chemist would like to determine how many mL of 6.2 M hydrochloric acid solution is needed to digest a 100 cm^3 sample of a common brass alloy ($Cu_{1.75}Zn$). The alloy has a density of 8.19 g/cm^3. In the digestion process HCl converts the Zn to $ZnCl_2$ and hydrogen gas. The Cu does not react with the acid.
IT IS CRITICAL THAT ALL UNITS BE EXPLICITY WRITTEN when solving these types of problems were we are converting between completely different units in most steps!

1. Did you read the problem carefully?
a) The starting unit is 100 cm^3 of an alloy.
b) The ending unit is mL of 6.2 M HCl solution.
c) Many conversion factors will be needed as well as a balanced chemical reaction!

2. The plan of conversion will be divided into several steps.
a) First we need to use the chemical formula of the brass alloy to find the mass % of Zn in the brass:
chemical formula brass → mass % Zn in brass

b) Then we use the stated volume of the brass sample to find the mass of brass and then use the mass % Zn to find the mass of Zn:
volume brass → mass brass → mass Zn

c) Then we need to balanced the chemical reaction:
? Zn(s) + ? HCl(aq) → ? $ZnCl_2$(aq) + ? H_2(g)

d) Using the mass of Zn in the sample we find the moles of Zn. Add the balanced reaction and we find the moles of HCl needed:
mass Zn → moles Zn → moles HCl

e) Finally, using the molarity of HCl we find the required volume:
moles HCl → mL HCl

3. Let's put the conversion factors and math to each step listed above.

a) Interpret the subscripts in the chemical formula ($Cu_{1.75}Zn$) as moles. Convert moles of each element to grams using molar mass:

$$(1.75 \text{ mol Cu})\left(\frac{63.55g}{1mol\ Cu}\right) = 11\underline{1}.2g\ Cu \text{ ; } (1.00 \text{ mol Zn})\left(\frac{65.39g}{1mol\ Zn}\right) = 65.\underline{3}9g\ Zn$$

Add the two masses to get the mass of one formula unit of brass. Use this to find the mass % Zn in the brass:
$11\underline{1}.2$g Cu + $65.\underline{3}9$g Zn = $17\underline{6}.6$ g brass.

$$\text{mass \% Zn} = \frac{65.\underline{3}9g\ Zn}{17\underline{6}.6g\ brass} x100\% = 37.\underline{0}3\%\ Zn$$

b) Now use the stated volume and density of the brass sample to find the mass of brass. Couple this with the mass % Zn and we have the mass of Zn in the 100 cm^3 brass sample:

$$100 \text{ cm}^3 \text{ brass}\left(\frac{8.19g\ brass}{1cm^3\ brass}\right)\left(\frac{37.\underline{0}3g\ Zn}{100g\ brass}\right) = 30\underline{3}.2g\ Zn$$

c) A balanced chemical reaction:
Zn(s) + **2** HCl(aq) → $ZnCl_2$(aq) + H_2(g)

d) Using the mass of Zn in the brass sample we find the moles of Zn. Add the coefficients from the balanced chemical reaction, and we find the moles of HCl needed:

$$30\underline{3}.2 \text{ g Zn}\left(\frac{1mol\ Zn}{65.39g\ Zn}\right)\left(\frac{2mol\ HCl}{1mol\ Zn}\right) = 9.2\underline{7}3mol\ HCl$$

e) Finally, using the molarity of HCl we find the required volume:

$$9.2\underline{7}3 \text{ mol HCl}\left(\frac{1L\ solution}{6.2mol\ HCl}\right)\left(\frac{1000mL}{1L}\right) = 1.50x10^3 mL\ solution = 1.50L\ solution\ .$$

4. Does this answer seem reasonable? Without experience you may not have a good feel for the magnitude of the correct answer. The 1.50 L of HCl does seems reasonable given 300 g of Zn is being digested.

05. Conversion Factors tutorial.doc

Summary

Conversion factors are used routinely in chemistry and physics. Mastering their use is essential for every student if they want to have a firm grasp of the material. This mastery comes from practice and recognizing that any time two quantities are related through and equals sign, xA = yB, or a ratio, $\frac{xA}{yB} or \frac{yB}{xA}$, you have been given a conversion factor. Follow the sequence of steps outlined in this tutorial rigorously until you have mastered using conversion factors, and REMEMBER - ALWAYS WRITE THE UNITS!

Self-Test

Complete the following problems. Round the final result to the correct number of significant digits. You will need a reference book to find any necessary equivalencies. Check your answers by reviewing the next page.

1. Convert 0.566 km to ft.

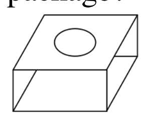
2. Convert 0.00195 oz to mg.
   (16 oz = 1 lb)

3. Find the volume in cm^3 of a room with dimensions 13.7 ft x 8.2 ft x 17.5 ft.

4. Find the body mass index (BMI) of a person that is 69.0 inches tall and has a mass of 158 lb. BMI has units of kg/m^2. If a persons BMI exceeds 25 kg/m^2 the person is considered overweight.

5. How many kg of methanol are needed to fill a 15.5-gal fuel tank? Methanol has a density of $0.791 g/cm^3$.

6. A square metal nut has the following dimensions: 14.0 mm on each edge, 6.0 mm thick, with a 7.0 mm diameter hole through the center. The density of the metal is 7.87 g/cm^3. Approximately how many nuts would be found in a 5.0-lb package?



Answers to Self-Test

1. Convert 0.566 km to ft

$$0.566\,\cancel{km}\left(\frac{0.6214\,\cancel{mi}}{1\,\cancel{km}}\right)\left(\frac{5280\,ft}{1\,\cancel{mi}}\right) = 1.86x10^3\,ft$$

2. Convert 0.00195 oz to mg
(16 oz = 1 lb)

$$0.00195\,\cancel{oz}\left(\frac{1\,\cancel{lb}}{16\,\cancel{oz}}\right)\left(\frac{453.6\,\cancel{g}}{1\,\cancel{lb}}\right)\frac{1000\,\cancel{mg}}{\cancel{g}} = 55.3mg$$

(I will not show the cancellation of units in the remaining problems.)

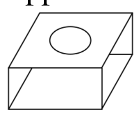
3. Find the volume in cm^3 of a room with dimensions 13.7 ft x 8.2 ft x 17.5 ft.
   a. Volume in ft^3 = 13.7 ft x 8.2 ft x 17.5 ft = $1.\underline{9}7x10^3\ ft^3$
   b. Linear conversion from ft to cm:
   $$ft\left(\frac{12in}{1ft}\right)\left(\frac{2.54cm}{1in}\right)$$
   c. Cubic conversion from ft^3 to cm^3:
   $$ft^3\left(\frac{12in}{1ft}\right)^3\left(\frac{2.54cm}{1in}\right)^3 = ft^3\left(\frac{1728in^3}{1ft^3}\right)\left(\frac{16.39cm^3}{1in^3}\right)$$
   d. Add the volume of the room in ft^3 and do the math:
   $$1.\underline{9}7x10^3\,ft^3\left(\frac{1728in^3}{1ft^3}\right)\left(\frac{16.39cm^3}{1in^3}\right) = 5.6x10^7\,cm^3$$

4. Find the body mass index (BMI) of a person that is 69.0 inches tall and has a mass of 158 lb. BMI has units of kg/m^2. If a persons BMI exceeds 25 kg/m^2 the person is considered overweight.
   a. Convert 69.0 in to m and then square:
   $$69.0in\left(\frac{2.54cm}{in}\right)\left(\frac{1m}{100cm}\right) = 1.7\underline{5}3m;\ (1.7\underline{5}3m)^2 = 3.0\underline{7}2m^2$$
   b. Convert lb to kg
   $$158lb\left(\frac{0.4536kg}{1lb}\right) = 71.\underline{6}7kg$$
   c. Find the BMI:
   $$\frac{71.\underline{6}7kg}{3.0\underline{7}2m^2} = 23.3\frac{kg}{m^2}$$. This person is not considered overweight, are you?

5. How many kg of methanol are needed to fill a 15.5-gal fuel tank? Methanol has a density of $0.791 g/cm^3$.
   a. Plan: 15.5 gal → Liters → cm^3 → g methanol → kg methanol
   (Notice the unit change from cm^3 to g methanol in conversion 3, that's where the density is needed.)

b. $$15.5gal\left(\frac{3.785L}{1gal}\right)\left(\frac{1000cm^3}{1L}\right)\left(\frac{0.791g\ methanol}{1cm^3}\right)\left(\frac{1kg}{1000g}\right) = 46.4kg\ methanol$$

6. A square metal nut has the following dimensions: 14.0 mm on each edge, 6.0 mm thick, with a 7.0 mm diameter hole through the center. The density of the metal is 7.87 g/cm^3. Approximately how many nuts would be found in a 5.0-lb package?



a. First find the volume of a nut without a hole in the center:
$14.0mm \times 14.0mm \times 6.0mm = 1\underline{1}76mm^3$

b. Find the volume of the hole in the center, $V_{cyl}=\pi r^2h$.
$$V_{cyl} = \pi(3.5mm)^2(6.0mm) = 2\underline{0}0mm^3$$

c. Take the difference to find the volume of the metal in one nut:
$1\underline{1}76mm^3 - 2\underline{0}0mm^3 = \underline{9}76mm^3$

d. Find the mass of one nut in lb using the volume and density:
$mm^3 \rightarrow cm^3 \rightarrow g\ metal \rightarrow lb\ metal$
$$\frac{\underline{9}76mm^3}{1nut}\left(\frac{1cm}{10mm}\right)^3\left(\frac{7.87g\ metal}{1cm^3}\right)\left(\frac{1lb}{453.6g}\right) = \frac{0.0\underline{1}7lb}{1nut}$$

e. Use the above mass per nut to find the number of nuts in a 5.0-lb bag:
$$5.0lb\left(\frac{1nut}{0.0\underline{1}7lb}\right) = \underline{3}00nuts$$