# Introduction

A few topics in Chemistry require the use of logarithms. This exercise is meant to be a review. For those students without prior knowledge in this area, there is no reason to be intimidated by logarithms. Most simply, logarithms are mathematical functions that extract the exponent from the exponential representation of a number. Antilogarithms (exponential functions) are literally functions that "undo" the taking of a logarithm.

There are two common logarithmic functions: logarithms to the base 10 ($\log_{10}$ or simply log) that are called common logarithms and logarithms to the base e ($\log_e$ or simply ln) that are called natural logarithms. We will discuss common logarithms initially because they are easier to explain and understand and then we will extend the explanation to natural logarithms.
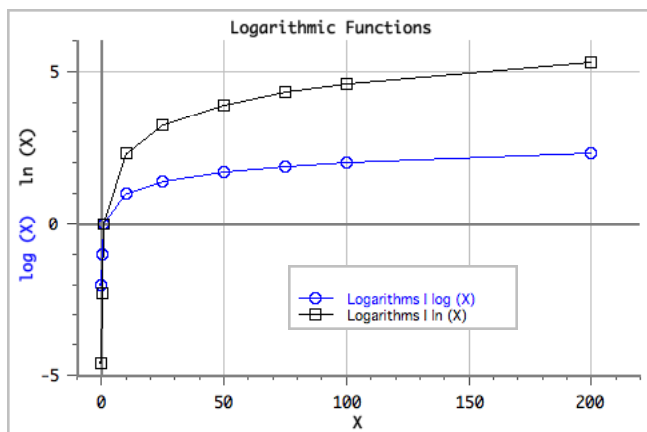
### Common Logarithms – log



Suppose we have a number $N = 10^X$. Then $\log_{10}N = X$, the log operator simply "extracted" the exponent from the base 10 exponential representation of the number. From this, it is easy to see that the antilogarithm of X is $10^X = N$. Taking logarithms of numbers which can be represented by integer powers of 10 is particularly easy, for example,

$\log 100 = \log 10^2 = 2$, $\log 0.001 = \log 10^{-3} = -3$, and

$\log 1 = \log 10^0 = 0$

Look at the graphs, notice the logarithmic functions change very rapidly for arguments (x values) less then 10 and then change much more slowly as x increases.

Evaluating antilogarithms of integers is even easier.

antilog $5 = 10^5 = 100000$ antilog $-2 = 10^{-2} = 0.01$ antilog $0 = 10^0 = 1$

**Note that although you cannot take the logarithm of a negative number or zero, you <u>can</u> take the antilogarithm of any real number.**

Taking the logarithm of a number that cannot be *wholly* represented by an integer power of 10 is less obvious. For example, you would have to know that $10^{0.3010} = 2.000$ in order to find

$$\log 2.000 = 0.3010.$$

In "the old days", it was necessary to use log tables to find the logarithms of such numbers. Fortunately, now all scientific calculators have "log" or "log x" functions to find the logarithm of any positive number within the computational range of the calculator. You should be able to find log 0.05 on your calculator (it's – 1.3) and log 3000 is 3.4771.

Finding the antilogarithm is just as easy. The antilog function on most calculators is activated by pressing $10^X$ key that is usually printed above the log key. You should be able to find the following antilogs by using your calculator:

antilog $2.6020 = 10^{2.6020} = 399.9$ antilog $-1.6990 = 10^{-1.6990} = 0.02000$

antilog $4.3010 = 10^{4.3010} = 2.000 \times 10^4$

### Natural Logarithms - ln

Use of natural logarithms (ln) is analogous to the use of common logarithms, except that the base of natural logs is the irrational number e (e = 2.71828...). This is a number like pi that arises naturally in mathematics and can't be represented by the ratio of any two integers (hence irrational). All scientific calculators have a natural log key usually labeled "ln" or "ln x", and an anti-natural log key which is usually labeled $e^x$ or exp() and is accessed in the same way as the $10^X$. Taking natural logs and antilogs is the same as taking common logs and antilogs except that one uses the "ln" and "$e^X$" instead of the "log" and "$10^X$" keys, respectively.

You should verify that you know how to find natural logarithms with the following:
ln 35 = 3.56     ln 10 = 2.30          ln 623.7 = 6.4357          ln $8.64 \times 10^{-2}$ = – 2.449
Also check that you can get the following results on your calculator:

antiln 1.36 = $e^{1.36}$ = 3.9 = exp(1.36)          antiln –0.362 = $e^{-0.362}$ = 0.696 = exp(-0.362)
antiln 0 = $e^{0}$ = 1 = exp(0)

In the natural sciences, many derivations result in relationships involving natural logarithms. For many years, before scientific calculators were widely available and inexpensive, common logarithms were found from tables or a slide rule. The natural logarithm was calculated from the base 10 logarithm using the formula: **ln(a) = 2.303 log(a)**.

### Mathematical Relationships of Logarithms and Exponential Functions

Sometimes it is necessary or desirable to manipulate logarithms. Some mathematical relationships involving logarithms are as follows. These relationships apply to logarithms of all bases.

| **Base 10** | **Base e** |
|---|---|
| $10^a \times 10^b = 10^{(a+b)}$ | $e^a \times e^b = e^{(a+b)}$ |
| $\log(10^a) = a$ | $\ln(e^a) = a$ |
| $10^{\log(a)} = a$ | $e^{\ln(a)} = a$ |
| $\log(ab) = \log(a) + \log(b)$ | $\ln(ab) = \ln(a) + \ln(b)$ |
| $\log(1/a) = -\log(a)$ | $\ln(1/a) = -\ln(a)$ |
| $\log(a/b) = \log(a) - \log(b)$ | $\ln(a/b) = \ln(a) - \ln(b)$ |
| $\log(a^n) = n \log(a)$ | $\ln(a^n) = n \ln(a)$ |
| $\log(a^{1/n}) = (1/n) \log(a)$ | $\ln(a^{1/n}) = (1/n) \ln(a)$ |

### Visualizing Exponential Functions

The functions $e^x$ = exp(x), $e^{1/x}$ = exp(1/x), $e^{-x}$ = exp(-x), and $e^{-1/x}$ = exp(-1/x) are very common in chemistry and physics. They are especially prevalent in the study of kinetics and thermodynamics. It is a good idea to recognize the shape of these functions and their limits for small and large values of x.



Comparison of Growth Functions

### $e^x$

$e^x$ can be evaluated for all real numbers x. The value of $e^x$ is always positive and greater than zero (green line). The graph shows $e^x$ approaches zero as x becomes more negative but increases rapidly

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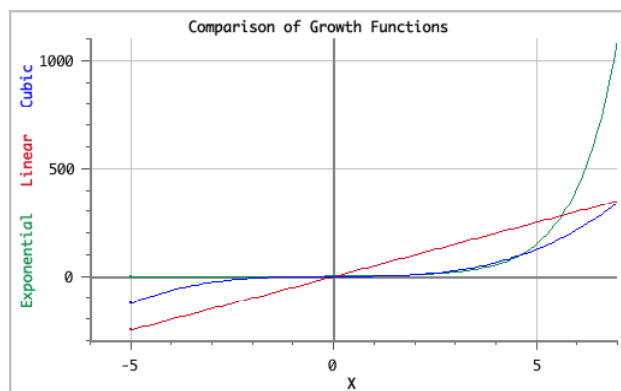
for positive values of x. This fast increase is called exponential growth. In nature, exponential growth is associated with uncontrolled population growth or the energy output from nuclear chain-reactions. Exponential growth is much faster than linear growth (red line) or cubic growth (blue line).

**$e^{-x}$**

The function $e^{-x}$ can also be written as $\dfrac{1}{e^x}$. This is simply the inverse of the $e^x$ function. The graph of

$e^{-x}$ is shown on the right for positive x values. This is an exponential decay curve. Many of the natural behaviors we observe in chemistry and physics follow this trend, first-order chemical or nuclear kinetics, heat flow between objects of different temperatures are two examples. (For negative values of x we simple have an exponential growth curve.)
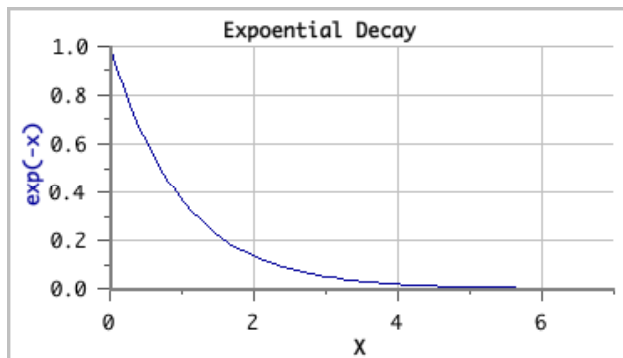


**$e^{1/x}$**

The function $e^{1/x}$ arises in chemistry when we discuss kinetics (Arrhenius equation) vapor pressure, (Clausius-Clapeyron equation) and thermodynamics. This function has a singularity for $x = 0$. For
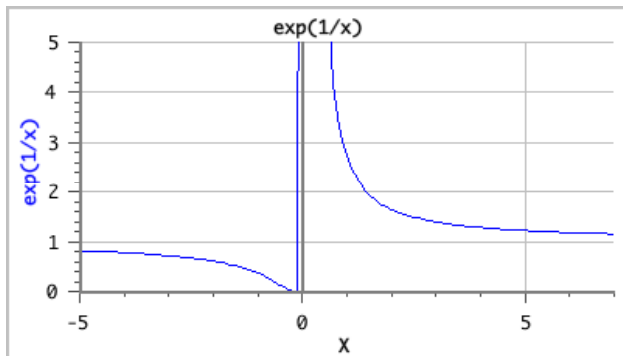


positive values of x, the function decreases rapidly as x moves away from zero and then slowly approaches the value of 1 for large x. The left side of the curve for negative values of x is simply the inverses of the right side of the curve. For large negative x the function approaches a value of 1 from below. As x approaches zero from the negative side the function approaches zero very rapidly. Since the limits of the function for negative x values are +1 and zero, we interpret this function as returning a "probability" of "fraction" when found in kinetic and thermodynamic equations.

**Significant Figures in Logarithms and Antilogarithms**
The rules for significant figures in logarithms are not as direct as one would like. See the tutorial "Significant figures in calculations" for the rules regarding logarithm and exponential functions.

## Self-Test

Answer the following questions. Check your answers by reviewing the next page.

Use your calculator to find the following values:
1. $\log(2.784)$
2. $\ln(0.00454)$
3. $e^{-3.786}$
4. $10^{23.888}$
5. $\log(2.3 \times 10^{-11})$
6. $\ln(5.4 \times 10^{-10})$

Use the properties of log and exponential functions to find the value of n.
1. $1.27 = 2^n$ (2 is exact)

2. $0.27 = \left(\dfrac{1}{2}\right)^n$ (1/2 is exact)

3. $1.2 \times 10^2 = 10^{-n}$
4. $2.88 = \ln(23.6 * n)$

5. $0.055 = e^{\frac{-3.3}{n}}$
6. $10.33 = \log(-4.5 \times 10^2 / n)$

In chemistry, the vapor pressure ($P$) of a liquid at Kelvin temperature $T$ can be determined from the following equation:

$$P(T) = e^{\left(-\frac{\Delta H^o}{RT} + \frac{\Delta S^o}{R}\right)}$$ where $\Delta H°$ (the heat of vaporization), $\Delta S°$ (the entropy of vaporization), and R (the gas constant) are all constants.

1. Use the properties of logs to derive an expression for $\ln(P)$.

    $\ln(P) =$

2. Given your answer to the previous question, use the properties of logarithms to derive the Clausius-Clapeyron equation:
$$\ln\left(\frac{P_2}{P_1}\right) = \frac{-\Delta H^o}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right);$$
Where $P_1$ is the vapor pressure at $T_1$ and $P_2$ is the vapor pressure at $T_2$.

## Answers to Self-Test

Use your calculator to find the following values:

1. $\log(2.784) = 0.0.4447$
2. $\ln(0.00454) = -5.395$
3. $e^{-3.786} = 0.0227$
4. $10^{23.78} = 6.0 \times 10^{23}$
5. $\log(2.3 \times 10^{-11}) = -10.64$
6. $\ln(5.4 \times 10^{-10}) = -21.34$

Use the properties of log and exponential functions to find the value of n.

1. $1.27 = 2^n$ (2 is exact)

    Take ln of both sides: $\ln(1.27) = \ln(2^n)$

    Properties of logs: $\ln(1.27) = n\ln(2)$

    Rearrange: $n = \dfrac{\ln(1.27)}{\ln(2)} = 0.345$

2. $0.27 = \left(\dfrac{1}{2}\right)^n$ (1/2 is exact)

    Take ln of both sides: $\ln(0.27) = \ln\left[\left(\dfrac{1}{2}\right)^n\right]$

    Properties of logs: $\ln(0.27) = n\ln\left(\dfrac{1}{2}\right)$

    Rearrange: $n = \dfrac{\ln(0.27)}{\ln\left(\dfrac{1}{2}\right)} = 1.89$

3. $1.2 \times 10^2 = 10^{-n}$

    Take log both sides: $\log(1.2 \times 10^2) = \log(10^{-n})$

    Properties of logs: $\log(1.2 \times 10^2) = -n$

    Rearrange: $n = -\log(1.2 \times 10^2) = -2.08$

4. $2.88 = \ln(23.6 * n)$

    Raise both sides to e: $e^{2.88} = e^{\ln(23.6 * n)} = 23.6 * n$

    Rearrange: $n = \dfrac{e^{2.88}}{23.6} = 0.75$

5. $0.055 = e^{\frac{-3.3}{n}}$

    Take ln both sides: $\ln(0.055) = \ln\left(e^{\frac{-3.3}{n}}\right)$

    Properties of logs: $\ln(0.055) = \dfrac{-3.3}{n}$

Rearrange: $n = \dfrac{-3.3}{\ln(0.055)} = 1.1$

6.  $10.33 = \log(-4.5 \times 10^2/n)$

Raise both sides to 10: $10^{10.33} = 10^{\log(-4.5 \times 10^2/n)} = -4.5 \times 10^2/n$

Rearrange: $n = \dfrac{-4.5 \times 10^2}{10^{10.33}} = -2.1 \times 10^{-8}$

In chemistry, the vapor pressure ($P$) of a liquid at Kelvin temperature $T$ can be determined from the following equation:

$P(T) = e^{\left(-\frac{\Delta H^o}{RT} + \frac{\Delta S^o}{R}\right)}$ where $\Delta H°$ (the heat of vaporization), $\Delta S°$ (the entropy of vaporization), and R (the gas constant) are all constants.

3.  Use the properties of logs to derive an expression for $\ln(P)$.

$$\ln(P) = \ln[P(T)] = \ln[e^{\left(-\frac{\Delta H^o}{RT} + \frac{\Delta S^o}{R}\right)}] = \frac{-\Delta H°}{RT} + \frac{\Delta S°}{R}$$

4.  Given your answer to the previous question, use the properties of logarithms to derive the Clausius-Clapeyron equation:

$$\ln\left(\frac{P_2}{P_1}\right) = \frac{-\Delta H^o}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right);$$

Where $P_1$ is the vapor pressure at $T_1$ and $P_2$ is the vapor pressure at $T_2$.

$\ln(P_2) = \dfrac{-\Delta H°}{RT_2} + \dfrac{\Delta S°}{R}$ and $\ln(P_1) = \dfrac{-\Delta H°}{RT_1} + \dfrac{\Delta S°}{R}$

Subtract $\ln(P_1)$ from $\ln(P_2)$;

$\ln(P_2) - \ln(P_1) = \dfrac{-\Delta H°}{RT_2} + \dfrac{\Delta S°}{R} - \left(\dfrac{-\Delta H°}{RT_1} + \dfrac{\Delta S°}{R}\right)$

Collect terms, the $\Delta S°/R$ term cancels;

$\ln(P_2) - \ln(P_1) = \dfrac{-\Delta H°}{RT_2} - \dfrac{-\Delta H°}{RT_1} = \dfrac{-\Delta H°}{R}\left(\dfrac{1}{T_2} - \dfrac{1}{T_1}\right)$

Properties of logs: $\ln(P_2) - \ln(P_1) = \ln\left(\dfrac{P_2}{P_1}\right) = \dfrac{-\Delta H°}{R}\left(\dfrac{1}{T_2} - \dfrac{1}{T_1}\right)$