Introduction

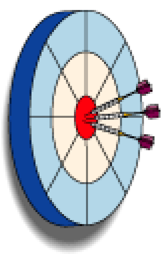
In this tutorial you will learn the definitions, rules and techniques needed to record measurements in the laboratory to the proper precision (significant figures). You should also develop a clear idea of how random errors determine the precision of a measurement and be able to work with these errors on an absolute and relative basis.

The objectives of this tutorial include

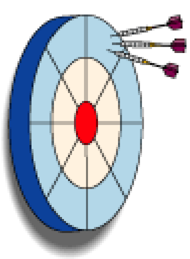
- To learn about types of errors associated with a measurement.
- To learn how to determine or estimate the precision (uncertainty) of a measurement.
- To work with relative and absolute errors confidently.

Why the fuss about measurements and precision?

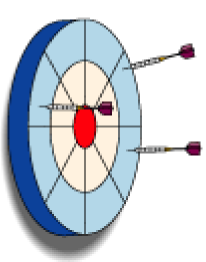
When a quantity is measured in the laboratory, that measurement comes with some uncertainty. This uncertainty reflects the precision of the measurement. Depending upon the instrument the uncertainty may be large relative to the measurement, like trying to measure the width of a hair using a mm scale! More often the uncertainty is small relative to the measurement, like one finds when measuring mass on an analytical balance. Regardless of the instrument the experimenter must be able to recognize the limitations of the measuring device and report all measurements to the correct precision.



Good accuracy
Good precision



Poor accuracy
Good precision



Poor accuracy
Poor precision

Measurements are a central part of the science of chemistry and physics. Any chemistry or physics textbook contains a large number of scientific facts and several scientific theories. The facts were obtained and the theories are supported by carefully made measurements. In order for experimental results to be meaningful, the experimenter must:

1. Record the results of his or her measurements carefully.
2. Repeat the measurement to increase its reliability. Since individual results from a series of repeated measurements will seldom be the same, a "best" value is obtained by taking the average, or mean, $\bar{x}$.
3. Establish the probable limits of precision that can be placed on the measurement.

When a measurement is made, one of the goals is to achieve an accurate and as precise a measurement as possible based on the instrument in use. The terms precision and accuracy are often used in discussing measured values. Precision **is an indication of how much uncertainty can be expected in a given measurement or the average of several measurements**. Accuracy refers to how closely a measurement agrees with the correct, or "true," value. The analogy of darts stuck in a dartboard illustrates the difference between these two concepts. **In lab we can always discuss the precision of a measurement, but *we cannot discuss accuracy unless the true value is known*.** In lab, the true value is often unknown!

Background

Errors associated with making measurements can be divided into two types: **systematic and random**. A **systematic error** causes a measurement to always be too low or too high. Systematic errors arise from a faulty instrument, a defect in the procedure or a consistent mistake in using the instrument. One example is an improperly calibrated balance. All systematic errors affect the accuracy of a measurement but not the precision. This tutorial will not discuss systematic errors and their prevention in any detail, but will focus on random errors, those errors that limit the precision of an instrument and determine the significant figures for a measurement.

Random errors result in individual measurements that are just as likely to be too high as too low. Random errors depend on the precision of the instrument and the skill of the person making the measurement. Random errors cannot be avoided, since there is always some degree of uncertainty in every measurement. Smaller random errors result in a higher degree of precision. As a final note, although high precision is often an indication of good accuracy, this need not always be true. It is possible to have low accuracy due to a systematic error, but still have high precision.

Precision (uncertainty) of a measurement

Each instrument (e.g., ruler, beaker, thermometer, balance, etc.) you use in the laboratory has a **precision** that determines the uncertainty (or error) in any measurement taken with that instrument. **The precision of an instrument is usually expressed in terms of a ± value.** In general, in a scientific measurement, the last digit recorded is taken to be inexact (it is estimated) and **is called the error digit**. Recording a measurement with the correct number of significant figures is necessary in order to reflect the inherent precision of the instrument. **Unless stated otherwise, the precision in a recorded measurement is taken to be ± one unit in the error digit.** For example, a reported value of 95.4 mL indicates to the reader a precision of ± 0.1 mL (95.4 ± 0.1 mL). If the precision in the measurement is not ± one unit in the error digit, then the experimenter has a responsibility to report the actual precision. If the precision is ± 0.5 mL the recorder should report 95.4 ± 0.5 mL. **A ± error, by definition, should only have one significant figure**. If we are uncertain about a volume in the first decimal place, we can know nothing about the uncertainty in the second, third, or any other decimal place!

Types of instruments – digital or analog.

The instruments you will use in lab can be divided into two types: those that have a digital scale and those that have an analog scale. Lets first discuss measurements and absolute precision from an instrument that has a digital scale, they are usually the easiest to use and interpret.

Precision of digital instrumentation – look it up!

When reading a digital scale it is best to refer to the manufacturers specifications as to the precision of the instrument. For example, consider a typical top-loading digital balance in a chemistry laboratory. According to the manufacturer the precision of a mass measurement depends upon the mass measured:

1. **If the mass is less than 100 g then the balance reads to 3 decimal places (mg) and has a stated precision of ± 0.001 g.**

2. **If the mass is greater than 100g the balance reads to 2 decimal places (cg) and has a stated precision of ± 0. 01 g.**

In this case the readout of the digital scale and the precision are the same, no erroneous digits are shown on the digital scale. The experimenter is confident the last digit displayed is the error digit. This may not always the case, making recording of digital measurements more complicated. For example, in chemistry we use digital thermometers attached to a LabPro data acquisition system. According to the manufacturer the precision of the thermometer is ± 0.02 °C, two decimal places. However, the digital readout shows four decimal places! The user must be careful to record the temperature to only two decimal places and disregard the two erroneous digits.

Precision of analog instrumentation – look it up or estimate from the scale.

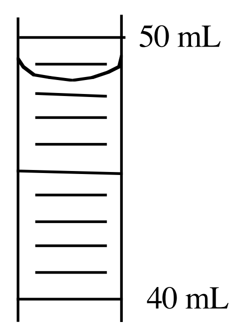
Instruments with analog scales fall into two types: 1) those that have a graduated scale and can make measurements over a range of values (e.g., ruler, thermometer, graduated cylinder, analog clock, etc) and 2) those that make a single measurement like a fixed volume of a liquid (i.e., volumetric glassware). Lets consider the single measurement instruments like volumetric glassware first and then instruments with graduated scales.

Volumetric glassware – single measurement devices

Consider the 10-mL volumetric pipet shown to the left. This will deliver 10 mL of liquid when filled to the calibration mark, but what is the precision? All volumetric glassware should have a stated precision on the device, if not you must refer to the manufacture's specifications. For the 10-mL pipet shown here the stated precision is ± 0.02 mL. Hence, when this 10-mL pipet is used correctly, the experimenter can expect the volume delivered to be between 9.98 and 10.02 mL on a consistent bases.

Graduated glassware and analog scales

Analog scales with graduations are treated differently than single measurement devices. With graduated scales **it is expected that the experimenter will estimate between the smallest scale division to make the measurement and determine the precision of the measurement.** The distance between graduations on a ruler, thermometer, buret or other glassware may be subdivided into ones, tenths, hundreds or other divisions depending on the precision of the instrument. A 50 mL graduated cylinder, for example (see illustration), has a smallest scale division of one mL. **Since the experimenter can estimate between the divisions, the volume can be measured and recorded to 1/10 mL, one decimal place**. A buret, on the other hand, has a smallest scale division of 1/10 mL. In this case the volume can be estimated to the 1/100 mL, two decimal places. Therefore, an extra digit of precision is gained when the buret is used over the 50-mL graduated cylinder. **For analog instruments, the precision is usually dependent upon the scale of the device.**



50-mL Graduated Cylinder Scale

The illustration shows the top part of a 50 mL graduated cylinder. The volume is read as 48.6 ± 0.2 mL. **The last digit (6) and the precision (± 0.2) are estimated by the experimenter**. The estimated precision indicates that the volume actually lies somewhere in the range of 48.4 to 48.8 mL.

A final note on scales and precision

Some analog instruments with scales may state the precision on the instrument. For example, a graduated pipet may state a precision of ± 0.03 mL even though the smallest scale division is 0.1 mL. **The stated precision will override any estimate the experimenter might make of the precision based upon the smallest scale division**.

Relative precision

In the previous discussion we have given the precision as an absolute value of ±x. Sometimes precision is given as a relative value to the measured quantity. For example, a 100-mL volumetric flask has an absolute precision of ±0.2 mL. The relative precision is $\frac{0.2mL}{100.0ml}x100\% = 0.2\%$.

Another example is a 50-ml graduated cylinder. When used to measure 25.0 mL the relative precision is: $\frac{0.2mL}{25.0ml}x100\% = 0.8\%$.

For an analytical balance the relative precision is extremely small for a 1-gram sample:

$$\frac{0.0001g}{1g}x100\% = 0.01\%.$$

Most laboratory beakers have a precision of ±5%. Thus, a 200-mL volume would have an absolute precision of ±10 mL.

Self-Test

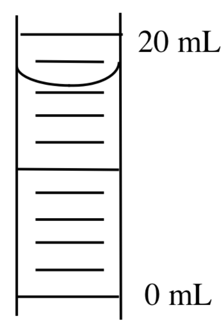
Answer the following questions. Check your answers by reviewing the next page.

True or False

1. Digital scales always display a measurement with the correct precision (significant figures).
2. The precision of an instrument should be available from the manufacturers specifications.
3. For an analog scale, the precision can usually be determined by estimating between the smallest scale division.
4. A precision of ± 0.03 mL implies the instrument is accurate to ±0.03 mL.
5. Precision can be expressed in absolute terms or as a relative term.
6. A systematic error will affect the precision of a measurement.

Problems

1. Use the illustration of a graduate glassware device to
   a. Read the level of the meniscus to the correct precision.
   b. Estimate the absolute precision of the measurement.
   c. Calculate the relative precision of the measurement.



2. A balance has a precision of ±0.05 g. A student obtains a mass of 3.42 g for a penny. Give the range of allowed values for the mass of the penny based on the precision.
3. A student reports a measured volume of 42.48 mL. As the reader of this volume, what absolute precision might you assign to this measurement?
4. A balance has a precision of ± 0.001 g. A sample that has a mass of about 25 g is placed on this balance. How many significant figures should be reported for this measurement?

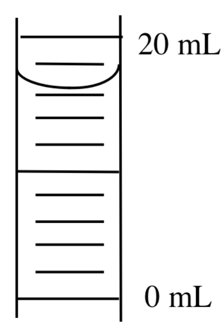
Answers to Self-Test

True or False

1. Digital scales always display a measurement with the correct precision (significant figures). FALSE. Some scales (LabPro for example) will display erroneous digits. In this case the experimenter must round the reading to the correct precision before recording. Digital balances ***never*** give erroneous digits!
2. The precision of an instrument should be available from the manufacturers specifications. TRUE. Most manufactures supply a specification sheet with the stated precision
3. For an analog scale, the precision can usually be determined by estimating between the smallest scale division.
   TRUE. Unless the precision of the instrument is known, the experimenter will determine the precision by how well the smallest scale division can be visually separated and the error digit estimated.
4. A precision of ± 0.03 mL implies the instrument is accurate to ±0.03 mL.
   FALSE. The precision is inherent to the instrument and only determines the uncertainty of a single measurement. Accuracy is determined by calibrating the instrument against a known standard. Although a measurement may be precise systematic errors will affect accuracy.
5. Precision can be expressed in absolute terms or as a relative term.
   TRUE.
6. A systematic error will affect the precision of a measurement.
   FALSE. Systematic errors affect accuracy, random errors affect precision.

Problems

1. Use the illustration of a graduate glassware device to
   a. Read the level of the meniscus to the correct precision.
      16.5 mL. The smallest scale division is 2 mL. The meniscus appears to be just above the 16 mL line but not quite to 17 mL.
   b. Estimate the absolute precision of the measurement.
      ±0.05 mL. That would divide the smallest scale division of 2 mL into 4 discernable divisions.
   c. Calculate the relative precision of the measurement.
      $$\frac{0.5mL}{16.5mL} x100\% = 3\%$$



2. A balance has a precision of ±0.05 g. A student obtains a mass of 3.42 g for a penny. Give the range of possible values for the mass of the penny based on the precision.
   The mass of the penny should be between 3.37 and 3.47 g.
3. A student reports a measured volume of 42.48 mL. As the reader of this volume, what absolute precision might you assign to this measurement?
   I would estimate al least ±1 unit in the error digit. That would be ±0.01 mL.
4. A balance has a precision of ± 0.001 g. A sample that has a mass of about 25 g is placed on this balance. How many significant figures should be reported for this measurement? The mass will be read to the third decimal place, 25.xxx g. There should be 5 significant figures.