Introduction

In this tutorial you will learn the definitions, rules and techniques for expressing numbers with the correct number of significant figures (digits). These rules will carry forward throughout your courses in chemistry and physics. Learn them well!

Why Significant Figures?

Expressing the proper significant figures in measured and calculated values conveys a sense of precision to the audience and defines a limit of error in the value. For example, if the mass of a penny is determined on an old triple beam balance, you might obtain a value of 3.15 grams, only 2 decimal places. However, with a modern analytical balance you might obtain 3.1474 grams, 4 decimal places! The second reading is much more precise than the first, hence the greater number of significant digits in the mass. Learning how to determine the correct number of significant figures in a number and how to round a number to the correct number of significant figures are the objectives of this tutorial.

A Few Definitions

1. Exact Numbers: NOT MEASURED!

These numbers have no uncertainty or error in their value, that is they are not measured but defined, and are taken to be exact. Examples are one dozen = 12, 1 liter = 1000 mL, etc. We will also assume throughout chemistry that all conversion factors that convert between like units (length to length, volume to volume, mass to mass) are exact. For example, 1 inch = 2.54 cm exactly. As a general rule, numeric definitions, most conversion factors, and whole numbers used for counting are taken to be exact. Note: measured physical properties such as density when used to convert between mass and volume are not exact.

2. Inexact Numbers: MEASURED VALUES!

When a value is determined by measurement, there is always some uncertainty in the measurement and the number is said to be INEXACT. For these inexact numbers, the number of significant digits will depend on the precision of the measurement and measuring device. For example, recall the mass of the penny from a triple beam balance, 3.15 grams. This measurement is inexact and contains 3 significant digits. Placing the penny on the modern analytical balance does not change its mass it only allows us to report the mass with greater precision, 3.1474 grams. This mass has more significant digits, 5, but is still an inexact value. As you may have guessed by now, measured quantities are NEVER EXACT, all come with some degree of uncertainty or error.

3. The error digit.

For an inexact number one of the digits is only estimated and therefore is uncertain, this is called the error digit. Digits before (to the left) of the error digit are certain, digits
after (to the right) of the error digit are erroneous and should be removed through rounding. When a number is written to the correct number of significant figures, the error digit will be the last digit written.

How do we determine which digit is the error digit in a measurement? The error digit is usually determined by the precision (scale) of the instrument used in making the measurement. This is the first digit in the measurement that is UNCERTAIN (not known exactly) and is estimated by the user (analog scale) or internally by the device (digital scale).

For example, a mass on a triple beam balance might be reported recorded as 3.15 g, three significant figures with the 5 being the error digit. Whereas the same mass on an analytical balance might read 3.1474 g, five significant figures with the right-most 4 being the error digit. In general, the greater the number of significant figures, the greater is the certainty implied for the measurement and therefore the greater is the precision of the measurement. When multiple measurements are made of a quantity, the results can be averaged, and the number of significant figures determined by using statistical methods. This method for determining the number of significant figures will be covered in a later tutorial.

Rules for Counting Significant Figures in an Inexact Number

1. All nonzero digits are significant: 1, 2, 3, 4, 5, 6, 7, 8, 9.
2. All embedded zeros are significant: 1203.
3. All leading zeros are not significant.
4. All trailing zeros are taken to be significant when written!
5. If the number is expressed in standard scientific notation, all digits in the coefficient (decimal portion) of the number are significant.

Examples. The correct number of significant figures is given in parenthesis with an explanation.

- a. 3.67 (3) All nonzero digits.
- b. 1.608 (4) Embedded zero is significant.
- c. 0.091 (2) Leading zeros are not significant, they are power of ten place holders.
- d. 0.0910 (3) Trailing zeros is significant since it is written.
- e. 1.30x10^{-3} (3) Decimal portion has three significant digits.
- f. 1.30x10^{3} (3) Decimal portion has three significant digits.
- g. 1300 (4) Trailing zeros are significant since they were written. Use scientific notation to eliminate insignificant trailing zeros. See below.

Rounding Numbers to the ERROR DIGIT

Often we will be required to round numbers to express the correct number of significant digits. There have been many different rules developed for rounding, the odd-even rule is one example. To keep things simple the round half-up rule will be applied. This is the method used by accountants and your calculator as well.

Rules for Half-up Rounding

1. If the number after the error digit is a 4 or less truncate to the error digit.
2. If the number after the error digit is a 5 or greater round the error digit up (half-up rule).
Examples. The following are rounded to the given number of significant figures (in parenthesis) with an explanation.

a. 1.3786 (3) → 1.38 To 3 significant digits the error digit is the 7. The number after the 7 is an 8. **Round the error digit up.**

b. 0.03403 (3) → 0.034 To 3 significant digits the error digit is the embedded 0. The number after the 0 is a 3. **Truncate after the error digit.**

c. 246.963 (4) → 247.0 To 4 significant digits the error digit is the 9. The number after the 9 is a 6. **Round the error digit up.** Notice the trailing zero is retained to give the number 4 significant digits.

d. -345.51 (3) → -346 To 3 significant digits the error digit is the first 5. The number after the 5 is also a 5. **Round the error digit up.**

e. 345.5 (3) → 346 To 3 significant digits the error digit is the first 5. The number after the 5 is a 5. **Round the error digit up.**

f. -345.5 (3) → -345 To 3 significant digits the error digit is the first 5. The number after the 5 is a 5. **Round the error digit up.** Since this is a negative number and the number after the error digit is **exactly** 5, rounding up means towards the greater (more positive) value.

g. 1345.2 (2) → 1300 → 1.3x10^3 To 2 significant digits the error digit is the 3. The number after the 3 is a 4. **Truncate to the error digit.** Since the truncated value is 1300, the trailing zeros are not significant (place holders) and must not be written. Use scientific notation to write with the correct number of significant digits.

The last example shows the usefulness of scientific notation when rounding numbers where the error digit occurs to the left of the ones place. When rounding these numbers insignificant trailing zeros will be introduced to maintain the proper magnitude for the rounded number. These insignificant trailing zeros should not be written. Converting the rounded number to standard scientific notation solves this problem.
Self-Test
Answer the following questions. Check your answers by reviewing the next page.

Determine if the following numbers or relationships are exact or inexact based on their use or context.

1. 12 inches = 1 foot.
2. A pencil measures 18.4 cm in length.
3. The number of marbles in a box is counted at 4567.
4. 1 calorie = 4.184 Joules
5. On Aug 12, 2009 the spot price of gold was $32.83/gram.
6. A container has a volume of 274.6 mL.

Count the number of significant figures in the following measured numbers.

1. 358 kg
2. 0.054 s
3. 6.3050 cm
4. 0.0105 L
5. 1.2348x10^{-4} m
6. -0.0340 atm

Round each of the following numbers to four significant figures, and express the result in standard scientific notation.

1. 102.53070
2. 656,980
3. 0.008543210
4. 0.000257850
5. -0.0357202
6. 345.805x10^{3}
Answers to Self-Test

**Determine if the following numbers or relationships are exact or inexact based on their use or context.**

1. EXACT. A definition.
2. INEXACT. A measurement
3. EXACT. A counting number, not a measured number.
4. EXACT. A conversion factor.
5. EXACT. A fixed price. It changes every minute but when quoted it is exact.
6. INEXACT. A measurement.

**Count the number of significant figures in the following measured numbers.**

1. (3)
2. (2)
3. (5)
4. (3)
5. (5)
6. (3)

In #6 the trailing zero counts since they were written.

**Round each of the following numbers to four significant figures, and express the result in standard scientific notation.**

1. 102.5 = 1.025x10^2
2. 657000 = 6.570x10^5
3. 0.008543 = 8.543x10^{-3}
4. 0.0002579 = 2.579x10^{-4}
5. -0.03572 = -3.572x10^{-2}
6. 345.805x10^3 = 3.458x10^5