Phy 4B 6/13

Tuesday, June 13, 2017 5:28 PM

Goals for the Lecture:

1) Be able to calculate inductance and EMF generated across an inductor
2) Understand that RL circuits are very similar to RC circuits
3) Be able to solve problems involving time constants for RL circuits
4) Be able to calculate energy stored in the magnetic field of an inductor

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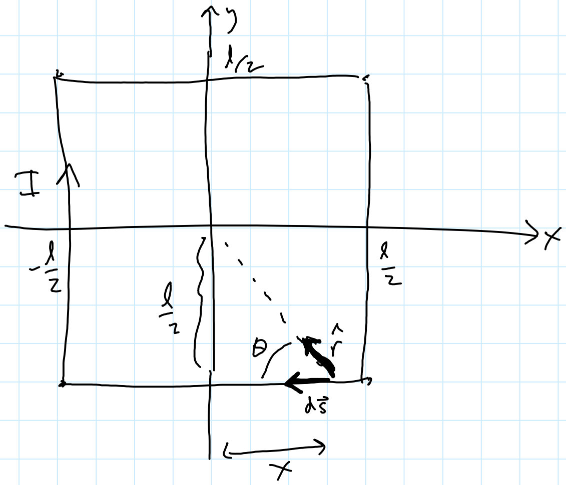
Review: Book Prob 30-5



$$B = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{s}\times\hat{r}}{\boxed{r^2}}$$

$$d\vec{s}\times\hat{r} = dx\,(1)\,\sin\theta$$

$$\sin\theta = \frac{\frac{l}{2}}{r} = \frac{l}{2\boxed{r}}$$

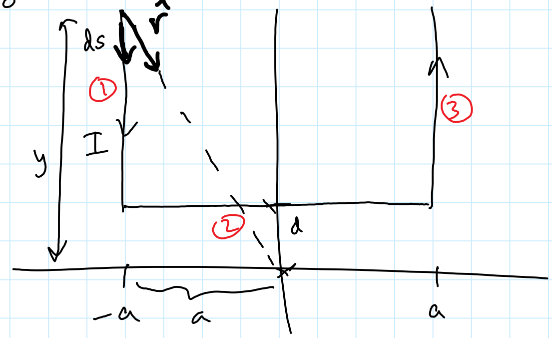
$$r = \sqrt{x^2+\left(\frac{l}{2}\right)^2}$$

$$B = \frac{8\mu_0 I}{4\pi}\int_0^{l/2}\frac{l}{2}\,\frac{dx}{\left[x^2+\left(\frac{l}{2}\right)^2\right]^{3/2}}$$

$$= \frac{8\mu_0 I}{4\pi}\,\frac{l}{2}\,\underbrace{\left.\frac{x}{\left(\frac{l}{2}\right)^2\sqrt{x^2+\left(\frac{l}{2}\right)^2}}\right|_0^{l/2}}_{\frac{\cancel{l/2}}{\left(\frac{l}{2}\right)^2\sqrt{2}\,\cancel{l/2}}}$$

$$= \frac{2\mu_0 I}{\pi\sqrt{2}}\,\frac{2}{l} = \frac{2\sqrt{2}\,\mu_0 I}{l\pi}$$

30-17



$$\vec{B}_{total} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

out of page

into page

out of page

$$B_1 = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{s}\times\hat{r}}{r^2}$$

$$d\vec{s}\times\hat{r} = dy\,(1)\,\sin\theta$$
$$= dy\,\frac{a}{r}$$
$$r = \sqrt{y^2+a^2}$$

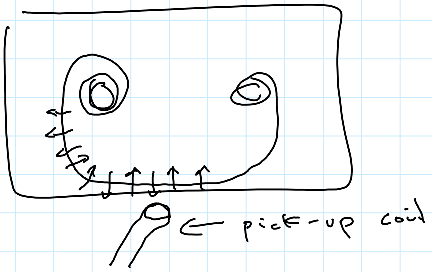
$$= \frac{\mu_0 I}{4\pi}\int_d^\infty \frac{a\,dy}{\left[y^2+a^2\right]^{3/2}}$$

$$= \frac{\mu_0 I a}{4\pi}\;\underbrace{\frac{y}{a^2\sqrt{y^2+a^2}}\Bigg|_d^\infty}_{\frac{1}{a^2\sqrt{\cancel{\frac{y^2}{y^2}}^{1}+\frac{a^2}{y^2}}}\Bigg|_d^\infty}$$

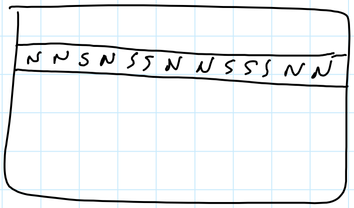
$$= \frac{\mu_0 I a}{4\pi}\left(\frac{1}{a^2} - \frac{d}{a^2\sqrt{d^2+a^2}}\right)$$

Applications.

cassette tape

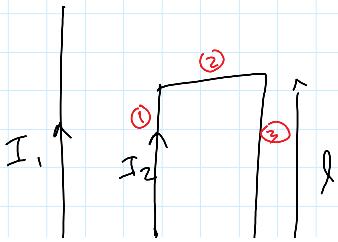


credit card



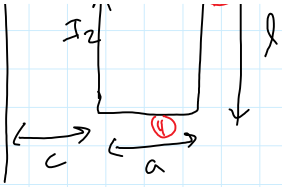
Speakers
Microphones

30-25



Find the Force on each side of the loop

direction of Forces:



direction of Forces:

F_1 ←

F_2 ↑

F_3 →

F_4 ↓

F_2 and F_4 will cancel

$$\vec{F}_2 = -\vec{F}_4$$

But, $F_1 > F_3$ so the

Net force is to the left

Let's find F_1:

1st find B from I_1 at the location of side ①

$$\int \vec{B} \cdot d\vec{s} = \mu_0 I_1$$

$$B\, 2\pi r = \mu_0 I_1$$

$$B = \frac{\mu_0 I_1}{2\pi r}$$

at $r = c$

$$B_1 = \frac{\mu_0 I_1}{2\pi c}$$

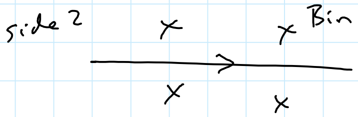
$$\vec{F} = I \vec{L} \times \vec{B}$$

$$F_1 = I_2\, l\, B_1$$

$$F_1 = \frac{I_2\, l\, \mu_0 I_1}{2\pi c} \quad \leftarrow$$

Find F_2:

$$B = \frac{\mu_0 I_1}{2\pi r}$$



$$\vec{F} = I \vec{L} \times \vec{B}$$

$\theta = 90°$

$\sin\theta = 1$

$$\int dF = \int I\, dl \times B$$

$$F = I L \times B$$

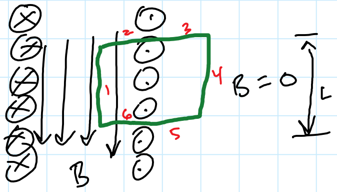
$$\int dF = \int I \, dL \times B \longleftarrow \sin\theta = 1$$

$$F = I_2 \int_c^{a+c} dr \, \frac{\mu_0 I_1}{2\pi r}$$

$$= \frac{\mu_0 I_1 I_2}{2\pi} \ln\left(\frac{a+c}{c}\right)$$

Ideal Solenoid

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{in}$$



$$\underbrace{\int_1 \vec{B}\cdot d\vec{s}}_{BL} + \underbrace{\int_2 \vec{B}\cdot d\vec{s}}_{\substack{0 \\ \vec{B}\cdot d\vec{s}=0 \\ B \perp ds}} + \underbrace{\underbrace{\int_3 \vec{B}\cdot d\vec{s}}_{0} + \underbrace{\int_4 + \int_5}_{0}}_{\substack{\text{because} \\ B=0}} + \underbrace{\int_6}_{\substack{0 \\ \vec{B}\perp d\vec{s} \\ \vec{B}\cdot d\vec{s}=0}} = \mu_0 I_{in}$$

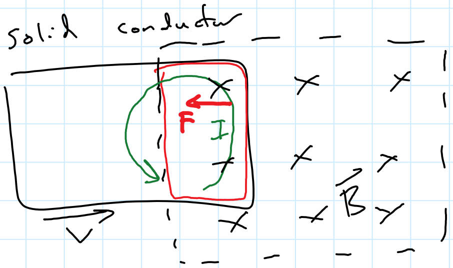
$$BL = \mu_0 I_{in}$$

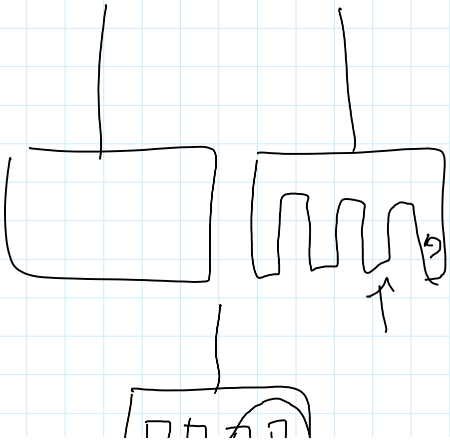
$$BL = \mu_0 N I$$

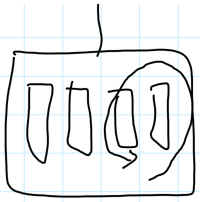
$$B = \mu_0 \frac{N}{L} I$$

$$B = \mu_0 n I$$

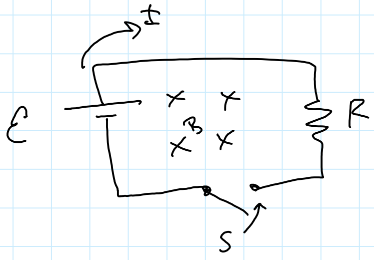
$$\uparrow \quad n = \frac{N}{L}$$







Inductance



1) close switch
2) current begins to flow
3) current creates a magnetic field (increasing magnetic flux into the page)
4) Increasing flux creates an EMF (or current) in the opposite direction (back EMF)

or Self - induction

Self - induced EMF:

$$\mathcal{E}_L = -N \frac{d\Phi_B}{dt}$$

for a solenoid:

$B = \mu_0 n I$ ideal solenoid

$\Phi_B = BA = \mu_0 n I A$

$$\mathcal{E}_L = -N \mu_0 n A \frac{dI}{dt} \left(\times \frac{I}{I} \right)$$

$$= -N \frac{\Phi_B}{I} \frac{dI}{dt}$$

$$= -L \frac{dI}{dt}$$

$N \quad \Phi$ Inductance

$$- \quad - \quad \frac{\quad}{dt}$$

For an ideal solenoid: $L = \frac{N \Phi}{I}$ Inductance

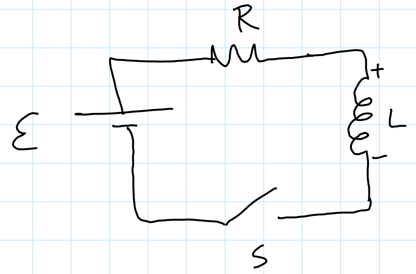
units for inductance: Henry $H = \frac{Vs}{A}$

Inductor: symbol: 

typically a large coil - like a solenoid

always opposes the change in current

RL Circuits



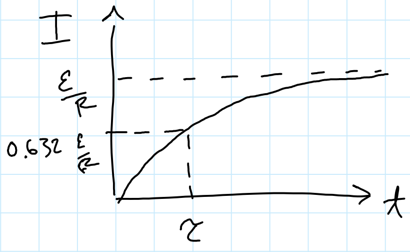
close switch at $t = 0$

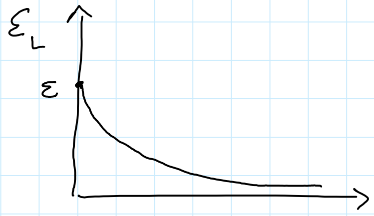
$$\mathcal{E}_L = -L \frac{dI}{dt}$$

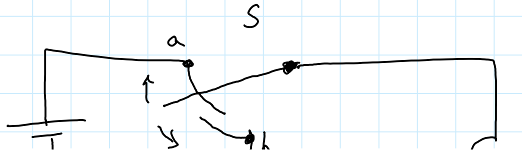
Loop Rule: $\mathcal{E} - IR - L \frac{dI}{dt} = 0$

$$I = \frac{\mathcal{E}}{R} \left(1 - e^{-\frac{t}{\tau}}\right)$$

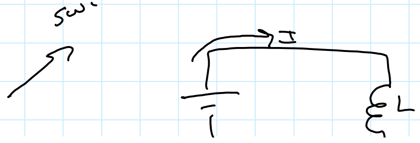
τ = time constant = $\frac{L}{R}$

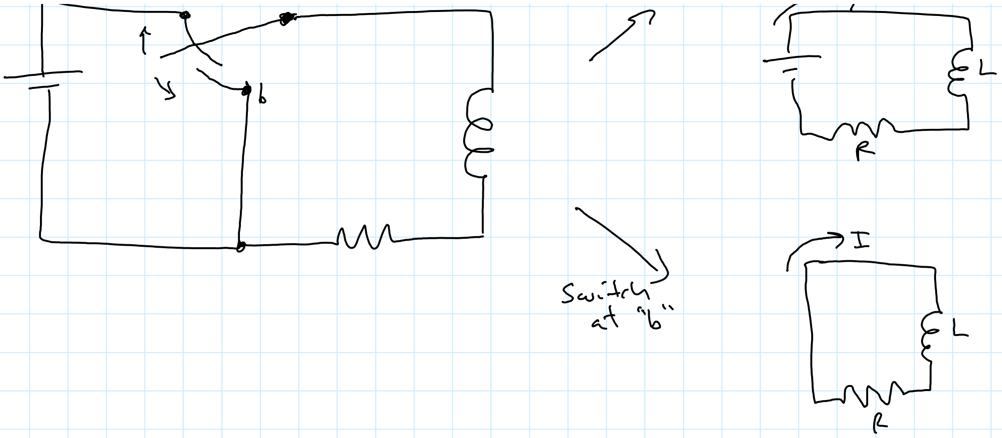




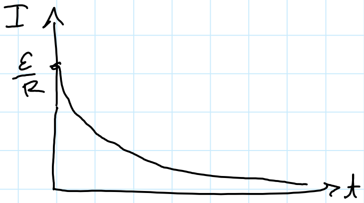


switch at "a"

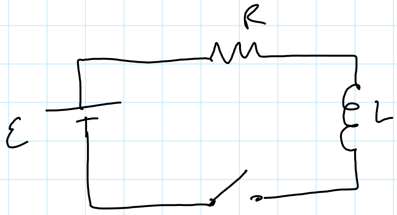




$$I = \frac{\mathcal{E}}{R} e^{-\frac{t}{\tau}}$$



Energy in the magnetic field:



Loop rule:

$$\mathcal{E} = IR + L\frac{dI}{dt}$$

multiply by I:

$$\underbrace{I\mathcal{E}}_{\text{Power supplied by battery}} = \underbrace{I^2R}_{\text{Power delivered to Resistor}} + \underbrace{LI\frac{dI}{dt}}_{\text{must be Power Stored in inductor}}$$

U = energy stored in inductor

$$\frac{dU}{dt} = \text{Power}$$

$$\frac{dU}{dt} = L I \frac{dI}{dt}$$

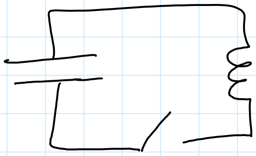
$$\boxed{U = \frac{1}{2} L I^2}$$ energy stored in inductor

Inductor → stores energy in magnetic field

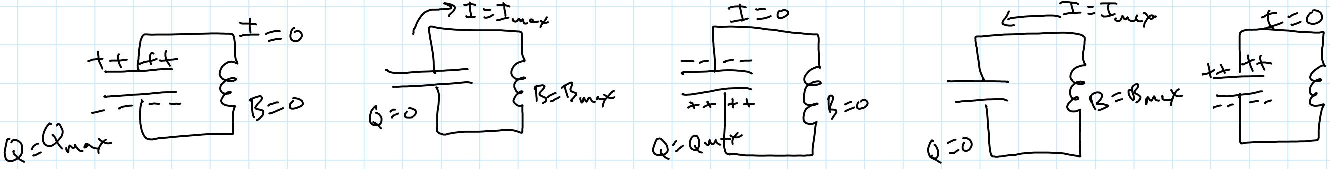
capacitor → " " " electric "

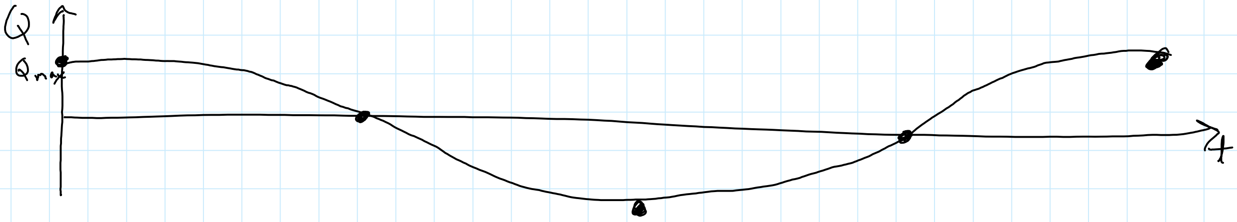
$$\boxed{U = \frac{1}{2} C V^2}$$ energy stored in capacitor

LC circuit



switch closed at $t=0$





$$\frac{d^2Q}{dt^2} = -\frac{1}{LC} Q$$

$$Q = Q_{max} \cos(\omega t + \phi)$$
↑

$$F = ma$$

$$-kx = m\, d^2x$$

$Q = Q_{max} \cos(\omega t + \phi)$

↑ sets initial condition

$$-\boxed{\frac{k}{m}}x = m\frac{d^2x}{dt}$$

if $F \propto x$

$\omega = \sqrt{\frac{k}{m}}$

$\omega = \frac{1}{\sqrt{LC}}$ angular freq of oscillation in an LC circuit