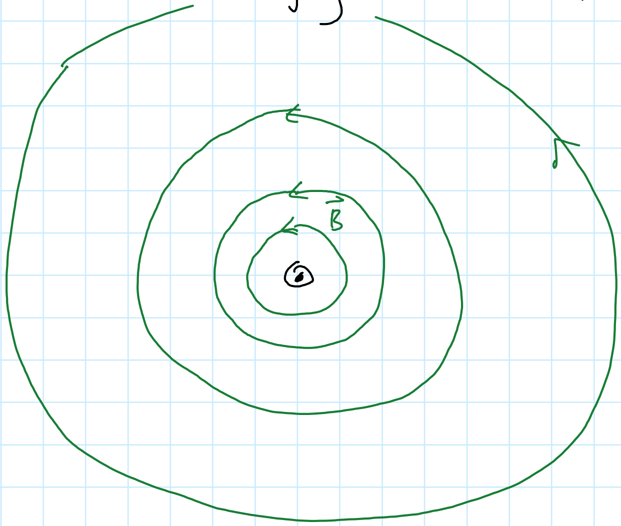
Phy 4B 6/1

Thursday, June 1, 2017 5:12 PM

Goals for the Lecture:

1) Be able to calculate the magnetic field from a current carrying wire segment using Biot-Savart Law (integrating over the current path)

Current carrying wire perpendicular to page



The magnetic field drops off as $\frac{1}{r}$ as you move away from wire

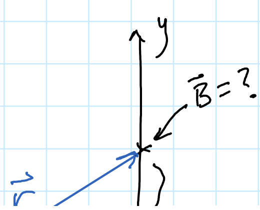
Biot-Savart Law: $$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$$

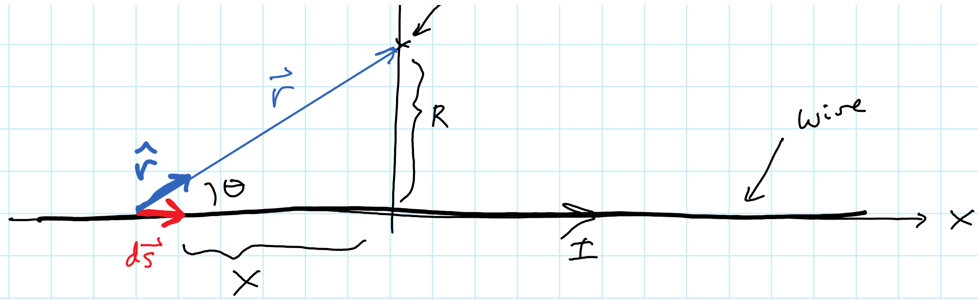
These are the same:

$$\frac{d\vec{s} \times \hat{r}}{r^2} = \frac{d\vec{s} \times \vec{r}}{r^3}$$

$$\mu_0 = 4\pi \times 10^{-7} \ \frac{T\,m}{A}$$

Example: Find the magnetic field a distance "R" from an infinite current carrying wire





$$B = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$$

$\mu_0 = 4\pi \times 10^{-7}$

I = current

$ds = dx$

$d\vec{s} \times \hat{r} = |d\vec{s}||\hat{r}| \sin\theta \quad \hat{k} \text{ or } \hat{z}$

$= dx \sin\theta$

$r = \sqrt{x^2 + R^2}$

$\sin\theta = \frac{R}{r}$

$$B = \frac{\mu_0 I}{4\pi} \int \frac{dx \ \sin\theta}{(x^2+R^2)}$$

$$= \frac{\mu_0 I}{4\pi} \int_{-\infty}^{+\infty} \frac{R \ dx}{(x^2+R^2)^{3/2}}$$

integrate over current path

$$= \frac{\mu_0 I}{4\pi} \, 2 \int_0^{\infty} \frac{R \ dx}{(x^2+R^2)^{3/2}}$$

$$= \frac{2\mu_0 I R}{4\pi} \left. \frac{x}{4R^2(x^2+R^2)^{1/2}} \right|_0^{\infty}$$

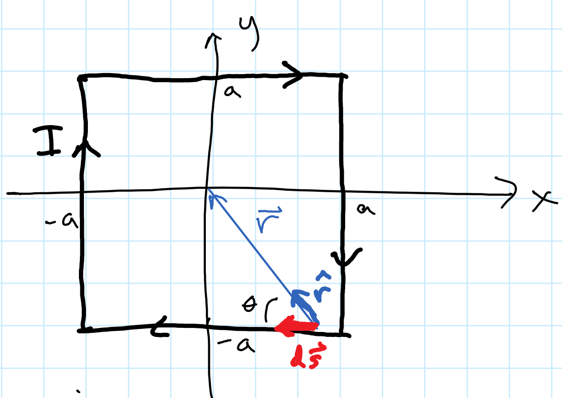
$$= \frac{\mu_0 I}{\ldots} \left. \frac{x}{\sqrt{\ldots}} \right|^{\infty}$$

$$= \frac{\mu_0 I}{2\pi R} \quad \underbrace{\frac{x}{\sqrt{x^2+R^2}} \Bigg|_0^{\infty}}_{1}$$

$$= \frac{\mu_0 I}{2\pi R}$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r} \quad \text{from a long, straight wire}}$$

Find B at the origin:



Square current loop

Length of each side is 2a

For one side:

$$B_1 = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$$

$$ds = dx$$

$$d\vec{s} \times \hat{r} = |d\vec{s}|\,|\hat{r}| \sin\theta$$

$$= dx\,(1)\,\sin\theta$$

$$= dx \left(\frac{a}{r}\right)$$

$\sin\theta = \frac{a}{r}$

$$B_1 = \frac{\mu_0 I}{4\pi} \int \frac{a}{r} \frac{dx}{r^2}$$

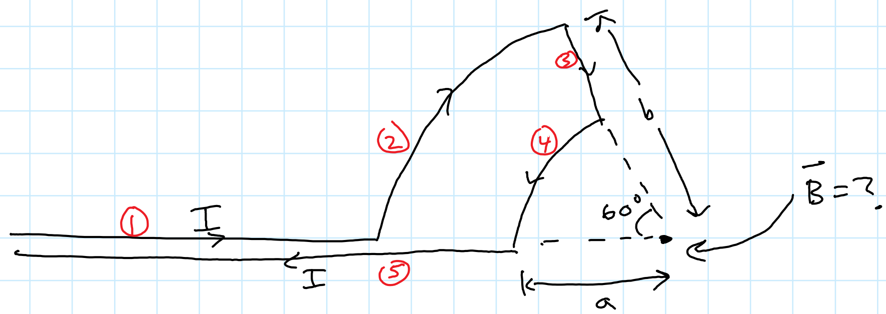
$$= \frac{\mu_0 I}{4\pi} a \int_{-a}^{a} \frac{dx}{(x^2+a^2)^{3/2}}$$

$$r = \sqrt{x^2+a^2}$$

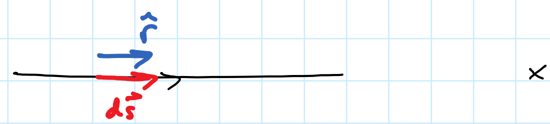
$$B_{total} = 4 \times B_1$$

one side

Example:



segment ①:

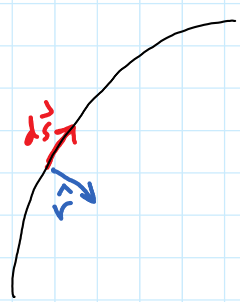


$$d\vec{s} \times \hat{r} = 0 \qquad \sin 0 = 0$$

③ $\quad d\vec{s} \times \hat{r} = 0$

⑤ $\quad d\vec{s} \times \hat{r} = 0$

②



$d\vec{s} \perp \hat{r}$

so, $\quad d\vec{s} \times \hat{r} = ds$

$$B = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2} \quad \text{into page}$$

$$= \frac{\mu_0 I}{4\pi} \int \frac{ds}{b^2}$$

$$= \frac{\mu_0 I}{4\pi} \frac{1}{b^2} \int ds \qquad \text{or} \qquad \int ds = \int_0^{\frac{\pi}{3}} r \, d\theta$$

$$= r \int d\theta$$

$$= \frac{\mu_0 I}{4\pi} \frac{1}{b^2} \, s \quad \text{length}$$

$$s = \frac{1}{3} \pi b \qquad = r \frac{\pi}{3}$$

$$= \frac{b\pi}{3}$$

$$= \frac{\mu_0 I}{} \quad \text{into page}$$

12 b

Segment ④ $B = \frac{\mu_0 I}{12 a}$ out of page

$$B_{total} = B_1 + B_2 + B_3 + B_4 + B_5$$

$$= 0 + \frac{\mu_0 I}{12 b} + 0 - \frac{\mu_0 I}{12 a} + 0 \quad (\text{into page is } +)$$

$$= \frac{\mu_0 I}{12}\left(\frac{1}{b} - \frac{1}{a}\right) \quad \text{into page is } +$$

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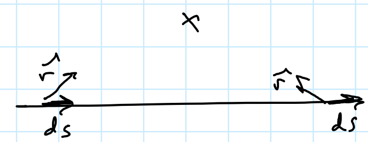
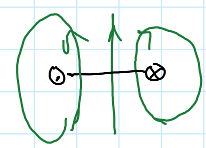
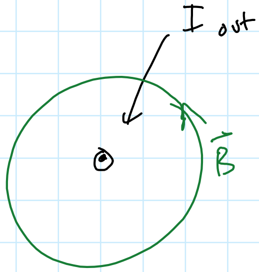
worksheet p. 119

Left	Right
out of page	out of page
Into page	↓
Out of page	out of page
out of page	Into page →





Ampere's Law



$B = \frac{\mu_0 I}{2\pi r}$

if we integrate B around the path:

$$\oint \vec{B} \cdot d\vec{s} = \oint B \, ds$$

$$= B \oint ds$$

$$= B \, 2\pi r$$

$$= \left(\frac{\mu_0 I}{2\pi r}\right)(2\pi r)$$

$$= \mu_0 I$$

for a circular path around a single wire

true for any path

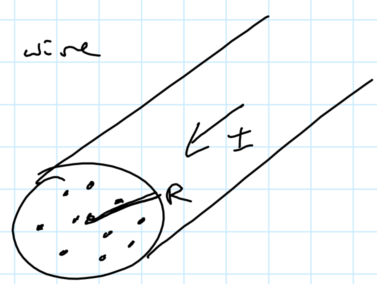
Ampere's Law : $\oint \vec{B} \cdot d\vec{s} = \mu_0 I$

I is total current enclosed by the path integral

useful for:

- infinite, straight wires
- solenoids

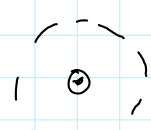
Example: A long, straight wire with current I uniformly distributed over cross section of wire



Find B everywhere:

1st) $r > R$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

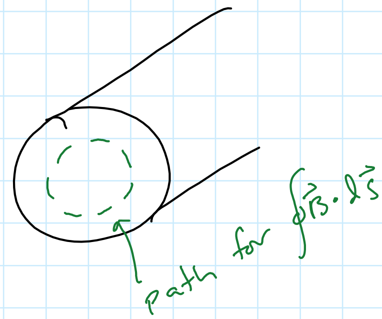


$$B \oint ds = \mu_0 I$$

$$B \, 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

2nd) $r < R$



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{in}$$

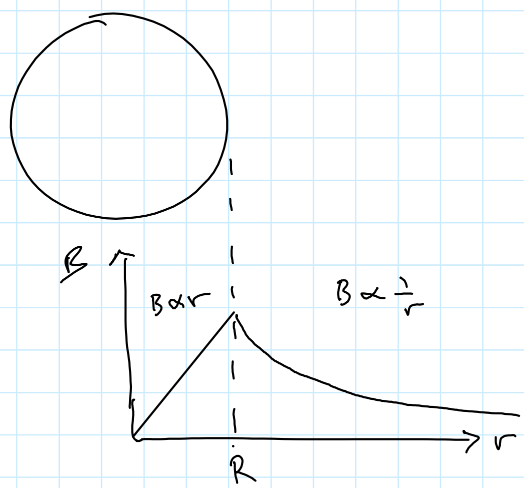
$$B \oint ds = \mu_0 I_{in}$$

$$B \, 2\pi r = \mu_0 I_{in}$$

$$B = \frac{\mu_0 I}{2\pi R^2} r$$

$$I_{in} = \frac{A_{in}}{A_{total}} I$$

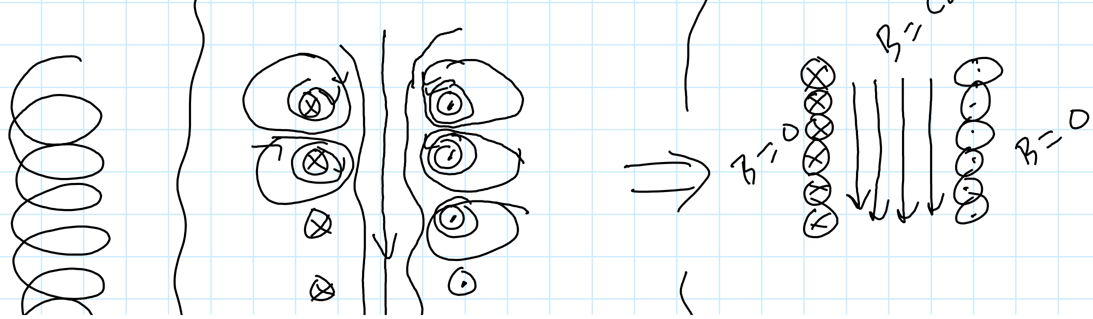
$$= \frac{\pi r^2}{\pi R^2} I$$

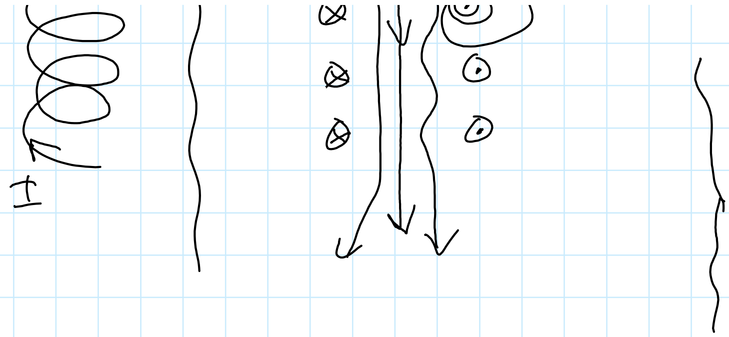


Solenoid: Coil of wire

cross section of coil

Ideal Solenoid

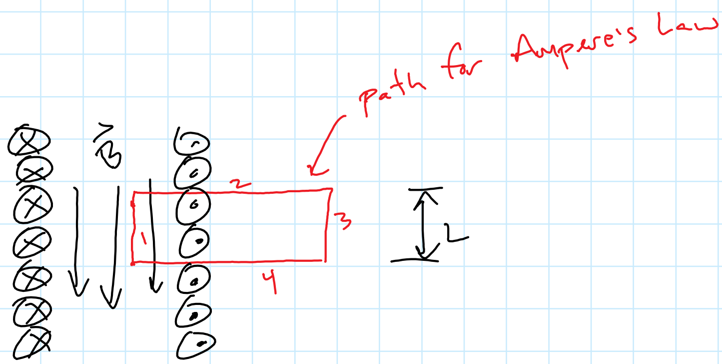




How to approach an ideal solenoid:
- turns are very close together
- length is much greater than radius

For ideal solenoid:
- internal field is uniform
- external field is zero

Find B inside an ideal solenoid: use Ampere's Law:



$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{in}$$

$$\underbrace{\int_1 \vec{B}\cdot d\vec{s}}_{BL} + \underbrace{\int_2 \vec{B}\cdot d\vec{s}}_{0} + \underbrace{\int_3 \vec{B}\cdot d\vec{s}}_{0} + \underbrace{\int_4 \vec{B}\cdot d\vec{s}}_{0} = \mu_0 I_{in}$$

B constant in same direction as ds

BL

since $\vec{B}\cdot d\vec{s} = 0$ or $B = 0$

$B = 0$

same as side 2

number of turns inside path

BL

$$BL = \mu_0 I_{in}$$

number of turns inside path

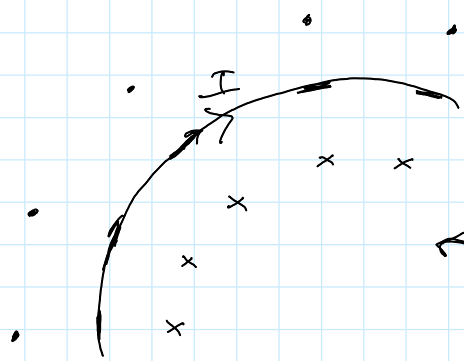
$$I_{in} = NI$$

$$BL = \mu_0 N I$$

$$B = \mu_0 \frac{N}{L} I$$

$$B = \mu_0 n I$$

$n = \frac{N}{L}$ turns per length



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

No nice path

use Biot-Savart

Magnetic Flux:

Electric flux: $$\Phi_E = \int_{surface} \vec{E} \cdot d\vec{A}$$

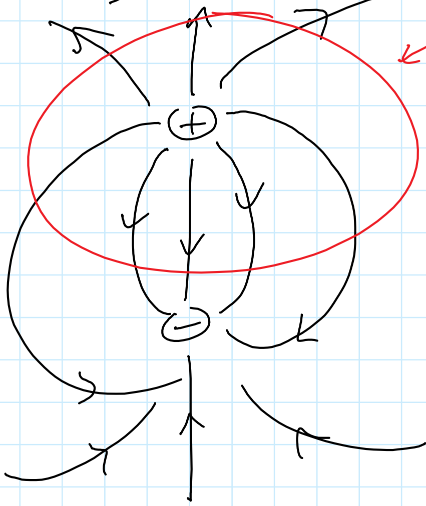
Magnetic flux: $$\Phi_B = \int_{surface} \vec{B} \cdot d\vec{A}$$

Gauss's Law: $$\Phi_E = \oint E \cdot ds = \frac{Q_{in}}{\epsilon_0}$$

charges

flux

charges



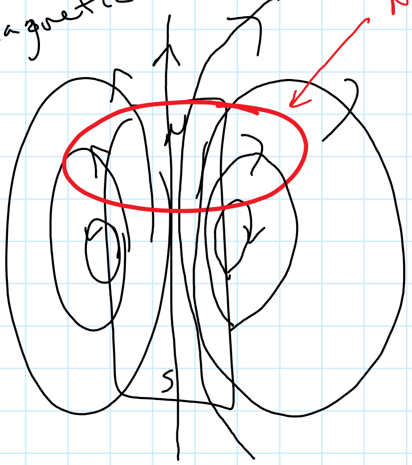
Net flux

Net charge enclosed

magnetic fields

No net flux

No magnetic monopoles



$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$

Gauss's Law for Magnetism : $\oint \vec{B} \cdot d\vec{A} = 0$

Ampere's Law : $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{in}$ is for finding B from currents

Ampere - Maxwell Law : $\oint \vec{B} \cdot d\vec{s} = \underbrace{\mu_0 I}_{\text{magnetic field created by a}} + \underbrace{\mu_0 \epsilon_0 \frac{d\Phi_E}{dt}}_{\text{magnetic field created by a changing}}$

field created by a current

created by a changing electric field

speed of Light = $\frac{1}{\sqrt{\mu_0 \epsilon_0}}$

Electro-magnet