

**Goals for the Lecture:**

- 1) Continue to solve electric potential problems using:

$$\Delta V = - \int_A^B \vec{E} \cdot d\vec{s}$$

And

$$V = \int \frac{k dq}{r}$$

- 2) Start capacitance

Worksheet  
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	$U_{int}$	$W_{on}$	$W_{By}$
1)	=	0	0
2)	=	0	0
3)	>	<	>
4)	<	>	<
5)	>	<	>
6)	>	<	>

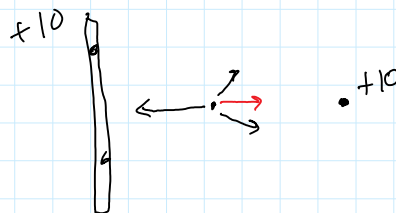
For a 3 particle system:

$$U = U_{12} + U_{23} + U_{13}$$

$$= \frac{k q_1 q_2}{r_{12}} + \frac{k q_2 q_3}{r_{23}} + \frac{k q_1 q_3}{r_{13}}$$

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top) To the left



Which direction will it move? | which direction will it move?

U

Bottom)

	Particle	Released From	Which direction is $\vec{E}$ ?	will it move?	which direction will it move?
1	+	A	+x	yes	+x
2	+	C	+x	yes	+x
3	+	E	-x	yes	-x
4	+	F	-x	yes	-x
5	-	B	+x	yes	-x
6	-	D	$E=0$	No	N/A
7	-	E	-x	yes	+x
8	-	F	-x	yes	+x

if  $\Delta V = - \int \vec{E} \cdot d\vec{s}$

then:  $E_x = - \frac{\partial V}{\partial x}$

$E_y = - \frac{\partial V}{\partial y}$

$E_z = - \frac{\partial V}{\partial z}$

in vector notation:  $\vec{E} = -\nabla V$

del  $\nabla = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$

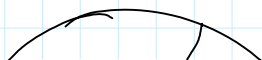
Book Prob

2 conducting, spherical shells:

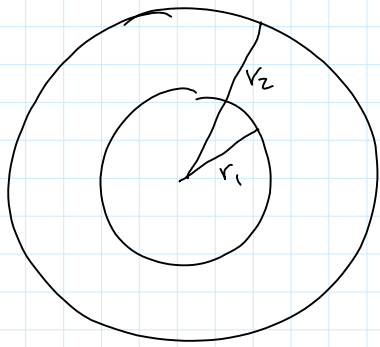
inner shell:  $r_1 = 15 \text{ cm}$   
 $q_1 = +10 \text{ nC}$

outer shell:  $r_2 = 30 \text{ cm}$   
 $q_2 = -15 \text{ nC}$

Find  $\vec{E}$  and  $V$  everywhere if  $V=0$  at  $r=\infty$



$V = \frac{kq}{r}$  For pt charge



$$V = \frac{kq}{r} \quad \text{For pt charge}$$

$$V = \int \frac{k dq}{r}$$

$$V = 0 \quad \text{at } r = \infty$$

1st) E

$$r < r_1$$

$$E = 0$$

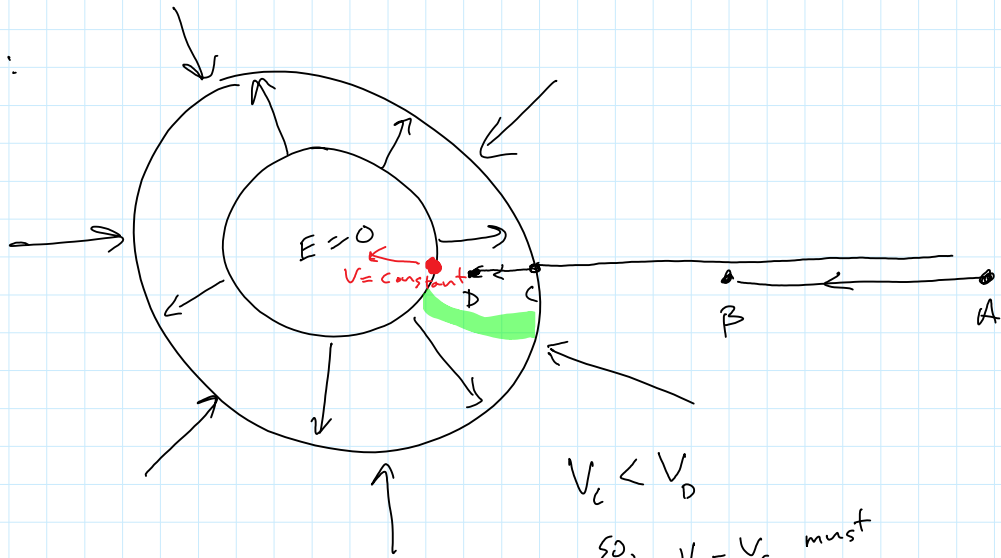
$$r_1 < r < r_2$$

$$E = \frac{k q_1}{r^2} \hat{r}$$

$$r > r_2$$

$$E = \frac{k (q_1 + q_2)}{r^2} = \frac{k (-5 \mu\text{C})}{r^2} \hat{r}$$

E fields:



2nd) V

start at  $V_A = \infty$

$$r > r_2$$

$$V_B - V_A = - \int_{\infty}^B \vec{E} \cdot d\vec{s}$$

$$V_B - V_A = - \int_{\infty}^B \frac{k(5\mu\text{C})}{r^2} dr (\cos 0^\circ)$$

must be negative  
( $V_B < 0$ )

$$= -k(5) \int \frac{dr}{r^2}$$

direction is accounted for here

$$\begin{aligned}
 (V_B < 0) &= -k(Q) \int \frac{dr}{r^2} \\
 &= +k(Q) \left. \frac{1}{r} \right|_{r_B}^{r_A} \\
 &= +k(5nC) \left( \frac{1}{r_A} - \frac{1}{r_B} \right) \\
 V_r &= -\frac{k(5 \times 10^{-9})}{r} \\
 &= -\frac{(8.99 \times 10^9)(5 \times 10^{-9})}{r} \\
 &= -\frac{45}{r}
 \end{aligned}$$

$$r_1 < r < r_2$$

Find  $V$  at  $r=r_2$

$$V_{r_2} = -\frac{45}{r_2} = -\frac{45}{0.3} = -150 \text{ V}$$

$$\begin{aligned}
 \Delta V &= -\int \vec{E} \cdot d\vec{s} \\
 V_D - V_C &= -\int_{r_b}^{r_c} \frac{kq_1}{r^2} dr (\cos 180^\circ)
 \end{aligned}$$

positive                      should be positive

$$\begin{aligned}
 &= kq_1 \int \frac{dr}{r^2} \\
 &= kq_1 \left[ -\frac{1}{r} \right]_{r_b}^{r_c} \\
 &= kq_1 \left( -\frac{1}{r_c} + \frac{1}{r_b} \right)
 \end{aligned}$$

positive ✓

$$\begin{aligned}
 V_D &= k(10 \times 10^{-9} C) \left( \frac{1}{r_b} - \frac{1}{r_c} \right) + V_C \\
 &= (8.99 \times 10^9)(10 \times 10^{-9}) \left( \frac{1}{1} - \frac{1}{1} \right) + (-150)
 \end{aligned}$$

$$= (8.99 \times 10^{11}) (6 \times 10^{-9}) \left( \frac{1}{r_0} - \frac{1}{0.3} \right) + (-150)$$

$$V_r = 90 \left( \frac{1}{r} - \frac{1}{0.3} \right) - 150$$

for  $r_1 < r < r_2$

for  $r < r_1$  Find  $V$  at  $r = r_1$

$$V_{r_1} = 90 \left( \frac{1}{r_1} - \frac{1}{0.3} \right) - 150$$

$$= 90 \left( \frac{1}{0.15} - \frac{1}{0.3} \right) - 150$$

$\underbrace{\hspace{1.5cm}}$   
 $\frac{1}{0.3}$

$$= 300 - 150$$

$$= 150 \text{ V}$$

$$V = 150 \text{ V} \quad \text{for } r < r_1$$

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Find  $E$  at each location:

$$E_A = 0$$

$$E_B = 0$$

$$E_C = \frac{kQ}{\left(\frac{3R}{2}\right)^2} = \frac{4}{9} \frac{kQ}{R^2}$$

$$E_D = 0$$

$$E_E = 0$$

$$E_F = \frac{8}{9} \frac{kQ}{R^2}$$

Find  $V$  at each location: (if  $V=0$  at  $r=\infty$ )

Find  $V$  at each location: (if  $V=0$  at  $r=\infty$ )

$$V_c = \frac{kQ}{\frac{3R}{2}} = \frac{2}{3} \frac{kQ}{R}$$

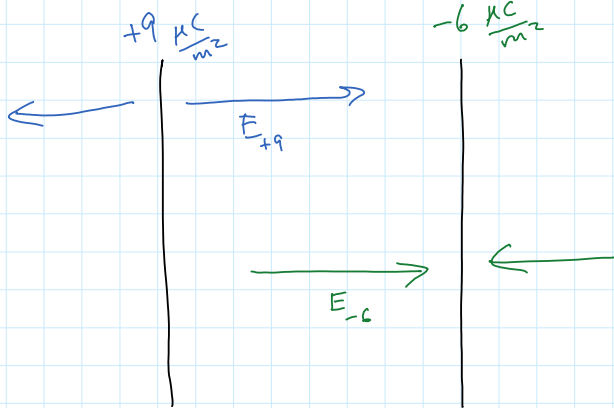
$$V_A = V_B = V_{\text{surface}} = \frac{kQ}{R}$$

$$V_F = \frac{k(2Q)}{\frac{3R}{2}} = \frac{4}{3} \frac{kQ}{R}$$

$$V_D = V_E = V_{\text{surface}} = \frac{k(2Q)}{R} = 2 \frac{kQ}{R}$$

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A)



$$E = E_{+9} + E_{-6}$$

$$E_{\text{sheet}} = \frac{\sigma}{2\epsilon_0}$$

$$= \left[ \frac{9}{2\epsilon_0} + \frac{6}{2\epsilon_0} \right] (\times 10^{-6})$$

$$= \frac{15 \times 10^{-6}}{2 (8.85 \times 10^{-12})}$$

$$\approx 1 \times 10^6 \frac{V}{m}$$

$$\Delta V = - \int \vec{E} \cdot d\vec{s}$$

$$= -Ed \quad \text{if } E \text{ is constant}$$

$$\Delta V = (1 \times 10^6) (0.08 \text{ m})$$

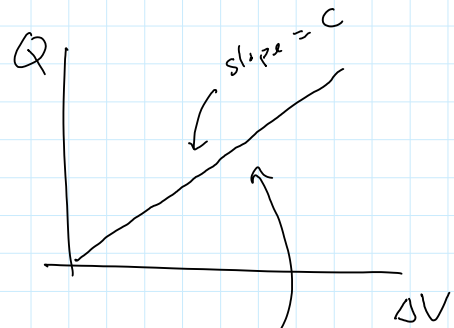
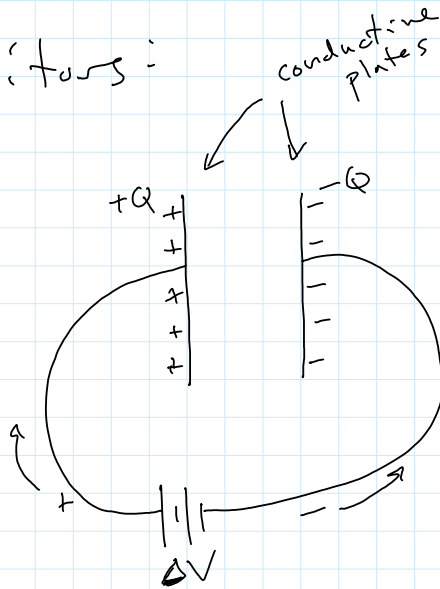
$$\approx 80,000 \text{ V}$$

Which plate is at higher potential?

Left

$\vec{E}$  points to Right  
 $E$  points to lower potential

Capacitors:



$$Q = C \Delta V$$

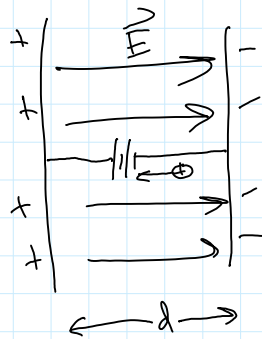
↑  
Capacitance

only depends  
on capacitor

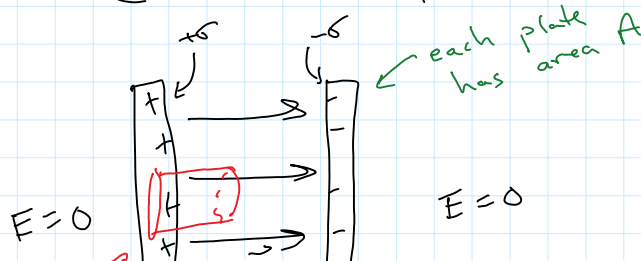
→ shape

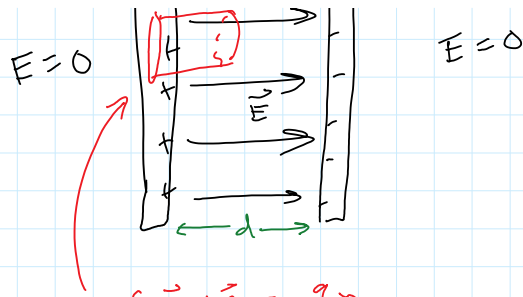
→ size

→ spacing



Find  $C$  for a parallel plate capacitor:





$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$\underbrace{\int_{\text{Left}} \vec{E} \cdot d\vec{A}}_0 + \underbrace{\int_{\text{Side}} \vec{E} \cdot d\vec{A}}_0 + \int_{\text{Right}} \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$E A = \frac{q_{in}}{\epsilon_0}$$

$$E = \frac{q_{in}}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

$\swarrow$  total charge  
 $\nwarrow$  total Area

$$|\Delta V| = \left| - \int \vec{E} \cdot d\vec{s} \right|$$

E is constant

$$\Delta V = \frac{Q}{\epsilon_0 A} d$$

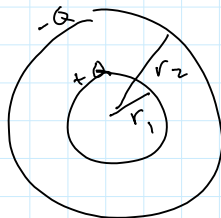
$$Q = C \Delta V$$

$$Q = \left( \frac{\epsilon_0 A}{d} \right) \Delta V$$

$$Q = C \Delta V$$

$$C = \frac{\epsilon_0 A}{d} \text{ for a parallel plate capacitor}$$

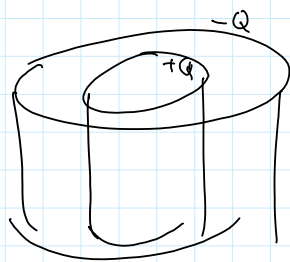
Find the capacitance of this capacitor:



2 concentric spherical shells  $r_1$  and  $r_2$

OR





2 concentric  
cylindrical shells