Phy 4B 5/2

Tuesday, May 2, 2017 4:08 PM

Goals for the Lecture:

1) Continue to solve electric potential problems using:

$$\Delta V = -\int_A^B \vec{E}\cdot\vec{ds}$$

And

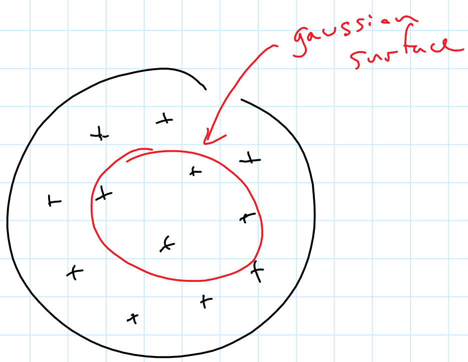
$$V = \int \frac{kdq}{r}$$

Solid insulating sphere w/ uniform charge distribution throughout its volume: [radius = R, total charge = $+Q$]

Find $\vec{E}$ and V everywhere (if $V=0$ at $r=\infty$)

1st) Find E everywhere using Gauss's Law

2nd) Use E in this equation: $\Delta V = -\int \vec{E}\cdot\vec{ds}$ to get V



Find E

$r > R$

$$\vec{E} = \frac{kQ}{r^2}\hat{r}$$

$r < R$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$EA = \frac{q_{in}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{q_{in}}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q_{in}}{r^2}$$

$$= k\frac{q_{in}}{r^2}$$

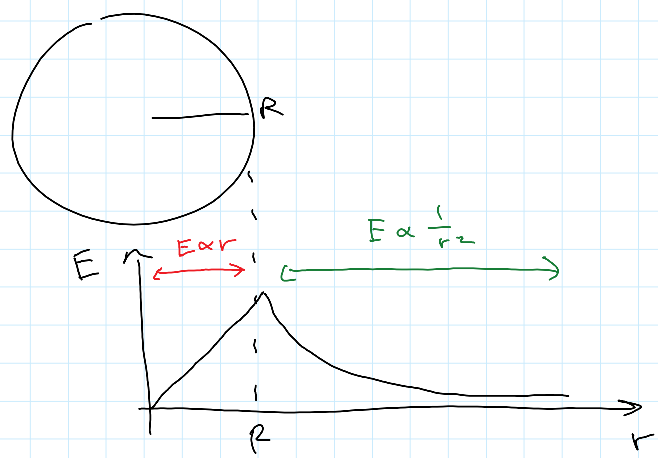
Find q_{in}:

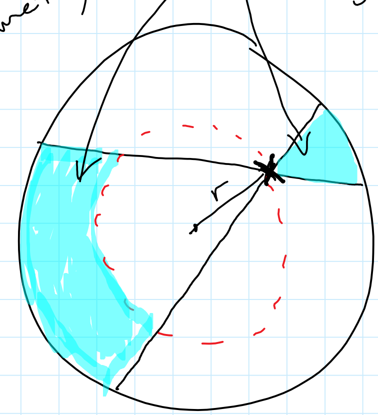
$$q_{in} = \frac{Q_{total}}{(Vol)_{total}}(Vol)_{inside}$$

$$= Q\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}$$

$$= \frac{Q r^3}{R^3}$$

$$E = \frac{kQ}{R^3} r \quad \hat{r}$$





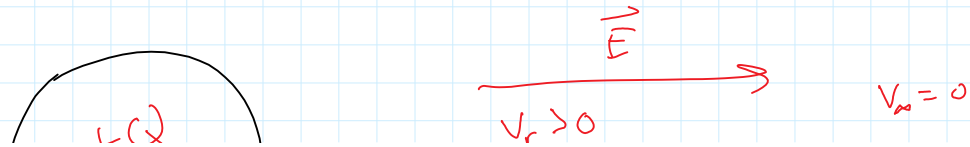
only charge inside your location contributes to $\vec{E}$

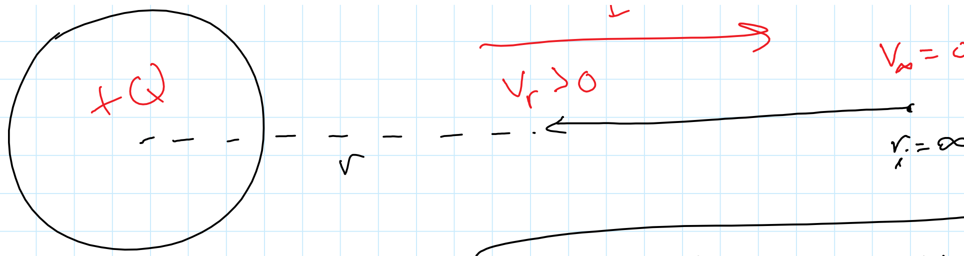
$$E = \frac{k\, q_{in}}{r^2}$$

Now Find V

$$\Delta V = -\int \vec{E} \cdot d\vec{s}$$

start where you know V → $V = 0$ at $r = \infty$





$$\boxed{V = \frac{kQ}{r} \quad \text{Just like a pt charge}}$$

$$\Delta V = -\int \vec{E} \cdot d\vec{s}$$

$$V_r - \cancel{V_\infty}^{\,0} = -\int_r^\infty \frac{kQ}{r^2}\, dr \,(\cos 180^\circ)$$

$$V_r = -kQ(-1)\int_r^\infty \frac{dr}{r^2}$$

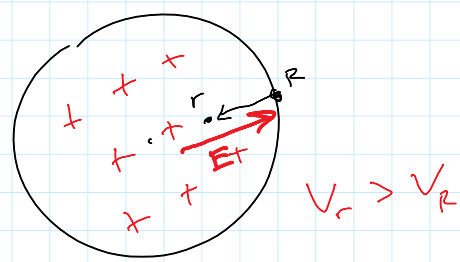
$$= kQ \left(-\frac{1}{r} \Big|_r^\infty \right)$$

$$\left[0 - \left(-\frac{1}{r}\right)\right]$$

$$\underbrace{V_r}_{\text{positive}} = \underbrace{\frac{kQ}{r}}_{\text{positive}} \quad \checkmark$$

<u>at surface $r = R$</u>

$$V_R = \frac{kQ}{R}$$



$$\Delta V = -\int \vec{E} \cdot d\vec{s}$$

$V_f - V_i$

$$V_r - V_R = -\int_r^R \frac{kQ}{R^3}\, r \, dr \,(\cos 180)$$

$$V_r - V_R = -\frac{kQ}{R^3}(-1)\int_r^R r\, dr$$

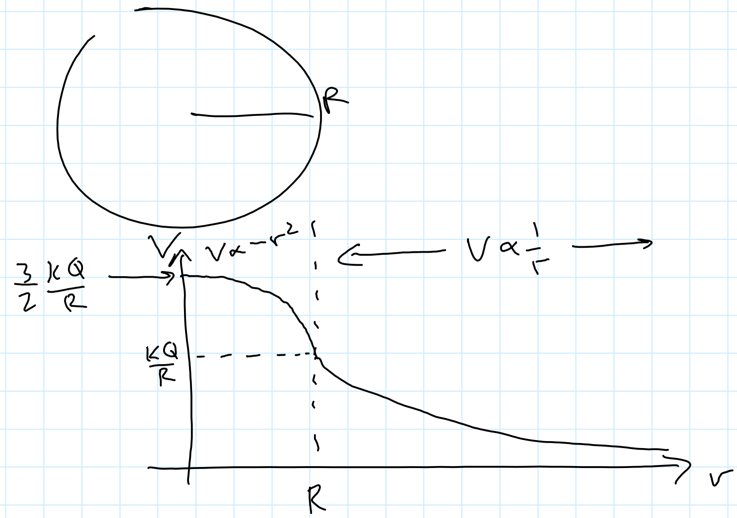
$$V_r - V_R = \frac{kQ}{R^3}\left(\frac{R^2}{2} - \frac{r^2}{2}\right)$$

$$\underbrace{V_r - V_R}_{+} = \underbrace{\frac{KQ}{R^3}}_{+} \underbrace{\left(\frac{R^2}{2} - \frac{r^2}{2}\right)}_{+}$$ ← check signs

we are good ✓

$$V_r = \frac{KQ}{2R^3}\left(R^2 - r^2\right) + \underbrace{V_R}_{\frac{KQ}{R}}$$

$$\boxed{V_r = \frac{KQ}{2R^3}\left(R^2 - r^2\right) + \frac{KQ}{R}}$$



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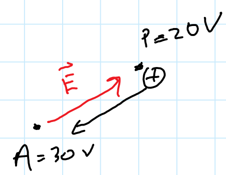
worksheet p. 65

$$W_{field} = -W_{ext}$$

A)



W_{field} is Negative

W_{ext} is positive

$$|W_{ext}| = |q\,\Delta V|$$

$$= |10\,q|$$

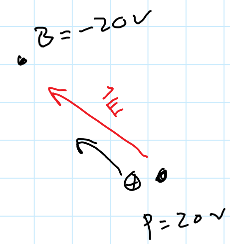
$$W_{ext} = +10q$$

B=−20V

B)

W_{ext}

B)



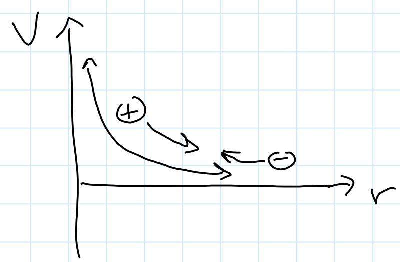
W_{field} is positive

W_{ext} is negative

$$|W| = |q \Delta V|$$
$$= |40q|$$
$$W_{ext} = -40q$$

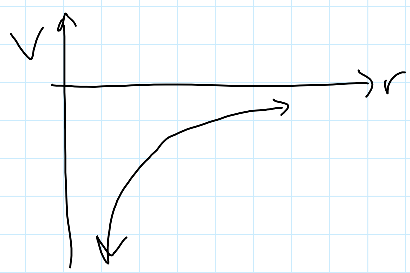
E) $V_P = V_E$ $\quad W = 0$

Potential from a pt charge: $+Q$



$$V = \frac{kQ}{r}$$

if $-Q$



$$V = -\frac{kQ}{r}$$

Potential Energy:

gravity → $F = mg \Rightarrow U_g = mgy$

gravity → $F = \frac{Gm_1m_2}{r^2} \Rightarrow U_g = -\frac{Gm_1m_2}{r}$

electric → E = constant

electric → $E \propto \frac{1}{r}$

electric → $E \propto \frac{1}{r}$

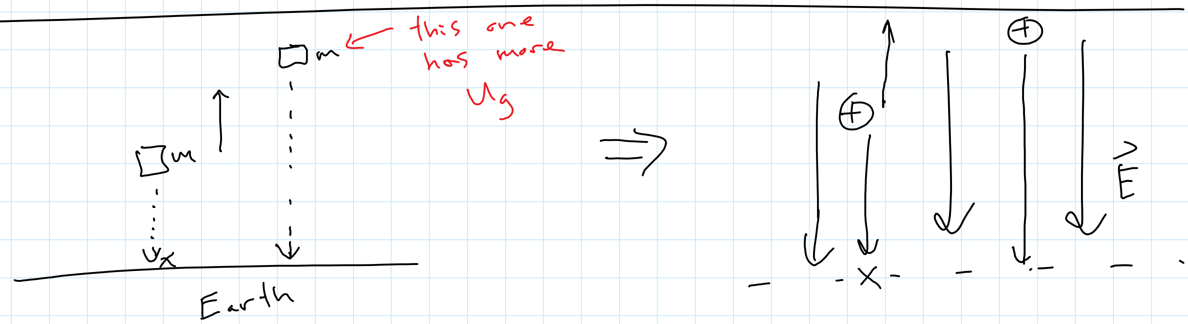
→ $E \propto \frac{1}{r^2}$

⋮

To find $U_{electric}$:

$\underline{W = -\Delta U}$ and $\underline{W = -q\,\Delta V}$ W done by field

potential energy



Potential Energy of a system of charges:

3 point charges: q_1, q_2, and q_3

1st) bring q_1 in from infinity

No work done

$U_1 = 0$

2nd) bring in q_2: Work is done against the field from q_1

$$U_2 = q_2 V_1 = \frac{k\,q_1 q_2}{r_{12}}$$

3rd) bring in q_3: Work is done by both fields (field from q_1 and field from q_2)

$$U_3 = q_3 (V_1 + V_2) = \frac{k\,q_1 q_3}{r_{13}} + \frac{k\,q_2 q_3}{r_{23}}$$

$$U_{system} = U_1 + U_2 + U_3$$

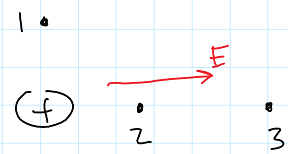
$$= \frac{k\, q_1 q_2}{r_{12}} + \frac{k\, q_2 q_3}{r_{23}} + \frac{k\, q_1 q_3}{r_{13}}$$

must use the correct sign for each charge

U can be + or −

Rank in order of increasing potential:

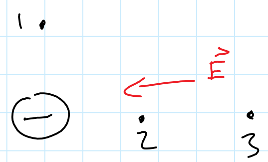
a)



$r_1 = r_2$

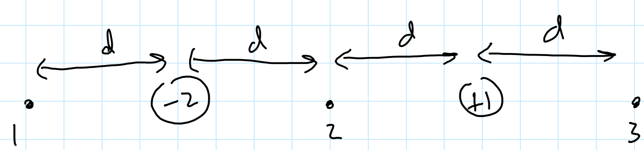
$\boxed{3 < 1 = 2}$

b)



$\boxed{1 = 2 < 3}$

c)



find V_1:

$$V_1 = V_{-2} + V_{+1}$$

$$= \frac{k(-2)}{d} + \frac{k(+1)}{3d}$$

$$= -\frac{5}{3}\frac{k}{d}$$

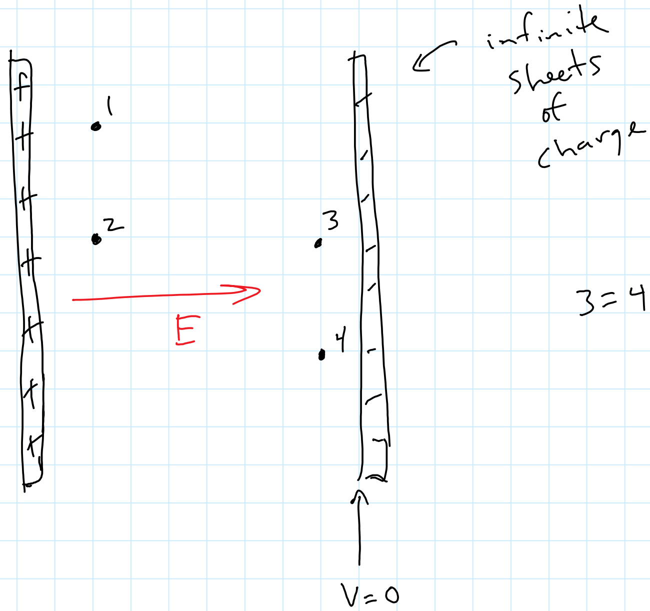
$$V_2 = V_{-2} + V_{+1}$$

$$= \frac{k(-2)}{d} + \frac{k(+1)}{d} = -\frac{k}{d}$$

$$V_3 = +\frac{1}{} \frac{k}{}$$

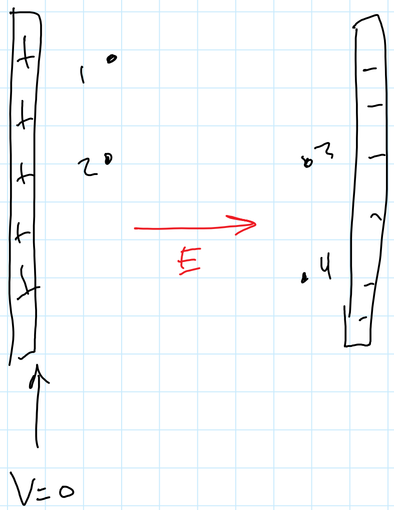
$\boxed{1 < 2 < 3}$

d)



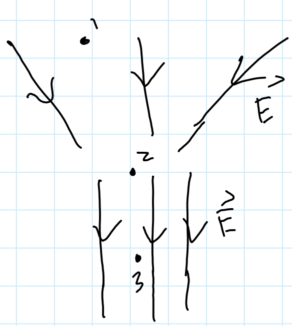
$3 = 4 < 1 = 2$

e)



$3 = 4 < 1 = 2$

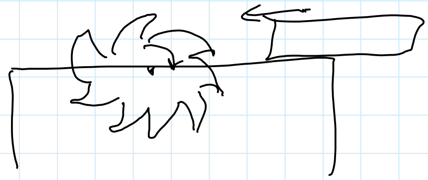
f)



$3 < 2 < 1$

Application: Saw stop

Application: Saw stop



Find E and V everywhere due to a charged (+Q) spherical conductor (radius R):

(Let $V=0$ at $r=\infty$)

Find E:

$\underline{r<R}$

$E=0$

$\underline{r>R}$

$E = \frac{kQ}{r^2}$

Find V:

$\underline{r>R}$

$V = \frac{kQ}{r}$

$\underline{r<R}$

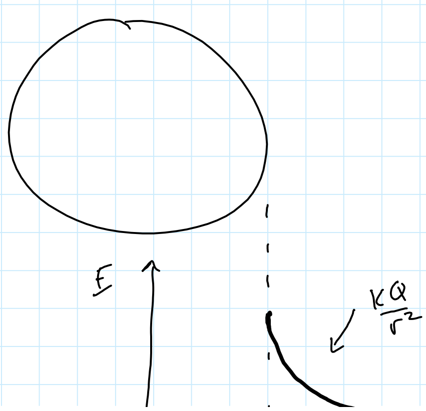
find $V_R = \frac{kQ}{R}$

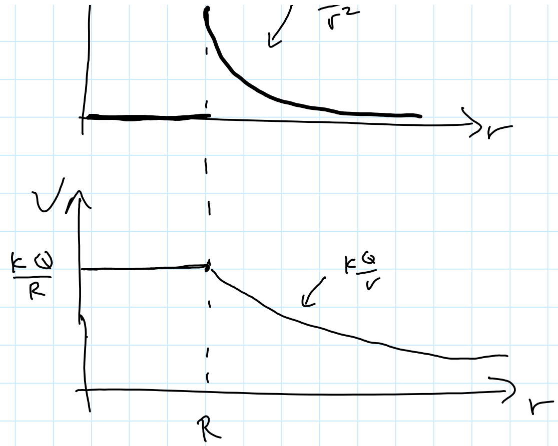
$\Delta V = -\int \cancelto{0}{\vec{E}} \cdot d\vec{s}$

$\Delta V = 0$

so, $V_r - V_R = 0$

$V_r = V_R = \frac{kQ}{R}$





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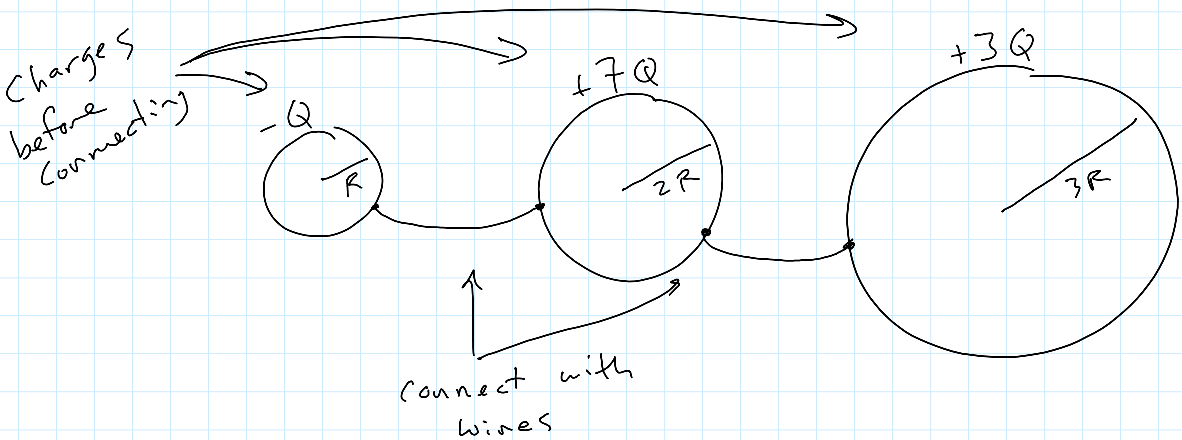
3 conducting, charged spheres:



find the charge on each after connecting them:

$$\boxed{V_R = V_{2R} = V_{3R}}$$

$$\frac{k\,q_R}{R} = \frac{k\,q_{2R}}{2R} = \frac{k\,q_{3R}}{3R}$$

$$q_R + q_{2R} + q_{3R} = -Q + 7Q + 3Q$$

$$= 9Q$$

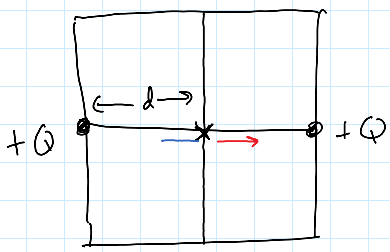
$$q_R + 2q_R + 3q_R$$

$$6q_R = 9Q$$

$$q_R = \frac{3}{2}Q$$

$$q_{2R} = 2\, q_R = 3Q$$

$$q_{3R} = 3\, q_R = \frac{9}{2} Q$$



Find $\vec{E}$ at the center:

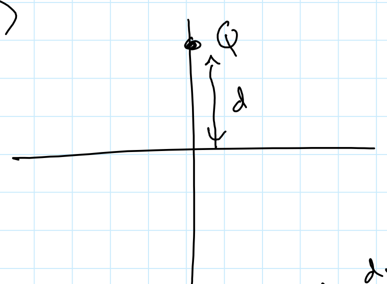
$$\vec{E} = 0$$

Find V at center:

$$V = \frac{kQ}{d} + \frac{kQ}{d} = \frac{2kQ}{d}$$

Find V at origin:

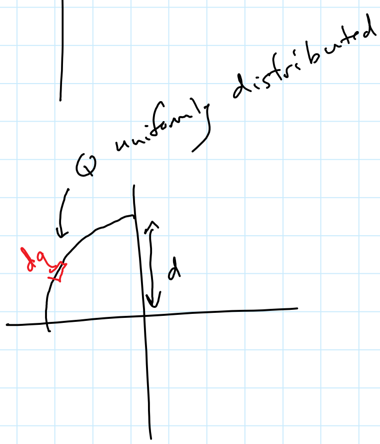
a)



$$\boxed{V = \frac{kQ}{r}}$$

$V = 0$ at $r = \infty$

b)

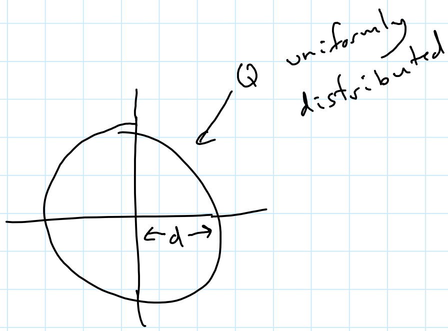


$$V = \int \frac{k\, dq}{d} = \frac{k}{d}\int dq = \frac{kQ}{d}$$

$$= \frac{k}{d}\int_0^{\pi/2} \lambda\, d\, d\theta$$

$$= k\lambda \frac{\pi}{2} = k\left(\frac{Q}{\frac{\pi d}{2}}\right)\frac{\pi}{2} = \frac{kQ}{d}$$

c)



$$V = \frac{kQ}{d}$$