Phy 4B 4/18

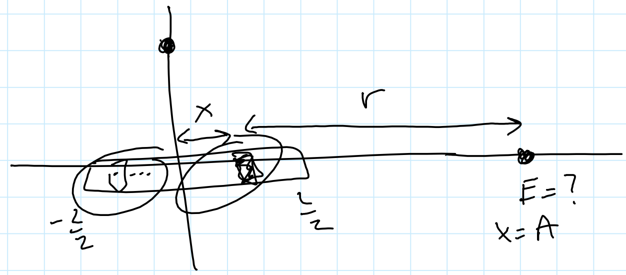
Tuesday, April 18, 2017 5:28 PM

Goals for the Lecture:

1) Understand electric flux and be able to calculate electric flux through a surface
2) Understand Gauss's Law and how it ties the electric flux to the charge enclosed

A couple of points about integrating to get E

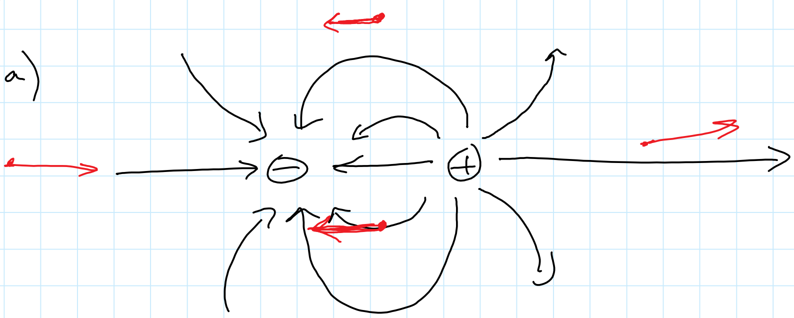
$$E = \frac{k q}{r^2}$$



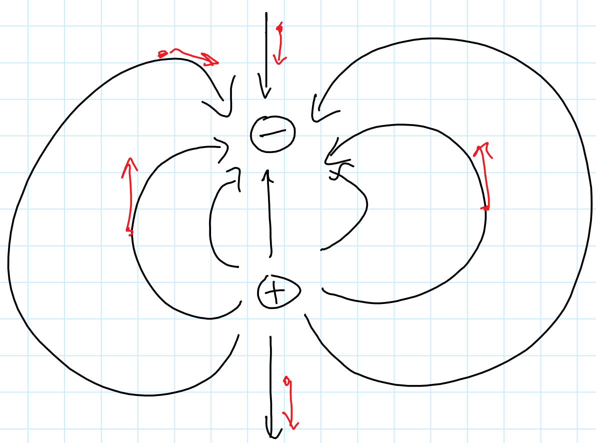
$$\int_{-L/2}^{L/2} \frac{k\,dq}{(A-x)^2}$$

worksheet p. 28

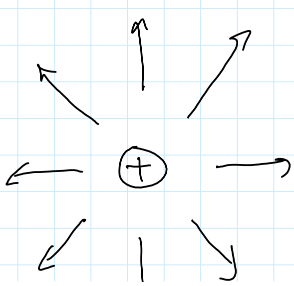
a)



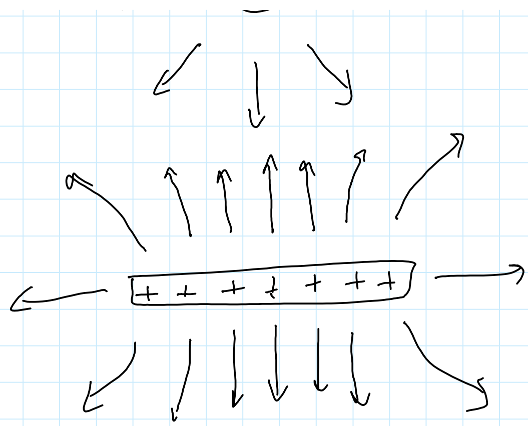
b)



c)



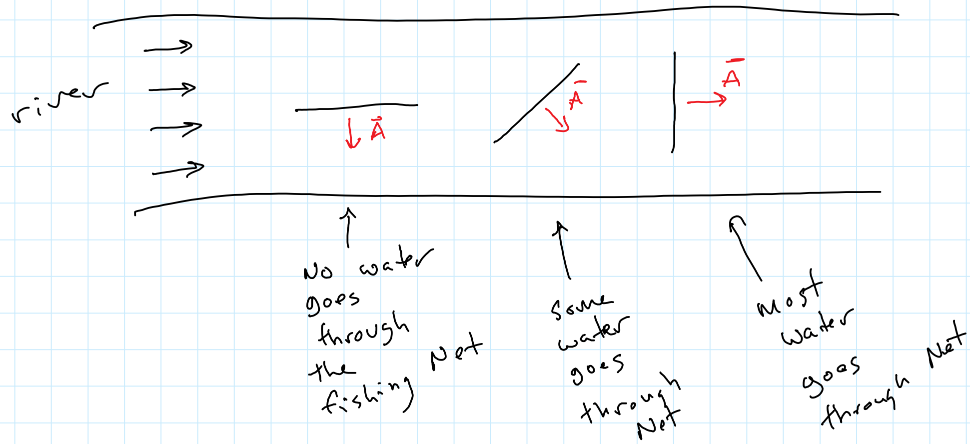
d)



Gauss's Law

Electric flux

Flux : fishing in a river



Variables

1) orientation of Net
2) flow of water
3) Area of Net

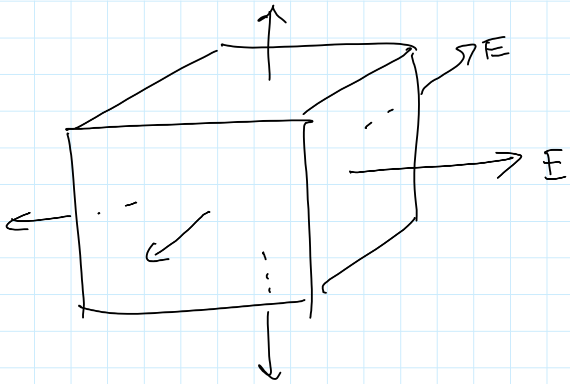
$$\Phi_E = EA\cos\theta \qquad \text{if } E \text{ is constant}$$

$$= \vec{E} \cdot \vec{A}$$

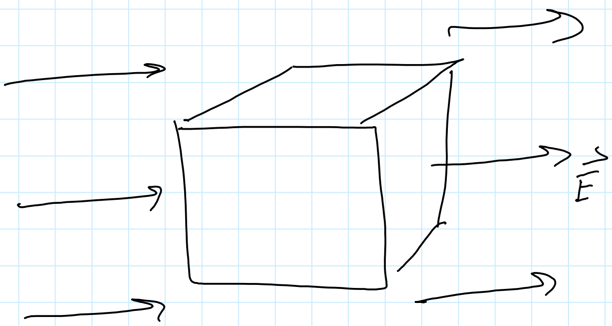
Area vector Normal to surface

In general:

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$



must have a net positive charge inside



Net charge inside is zero

Gauss's Law:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

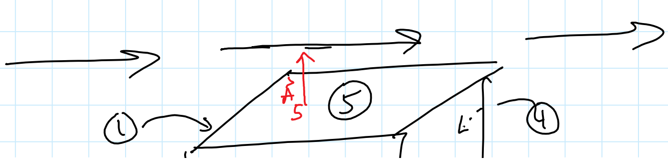
$$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N^2 m^2}$$

To find E:

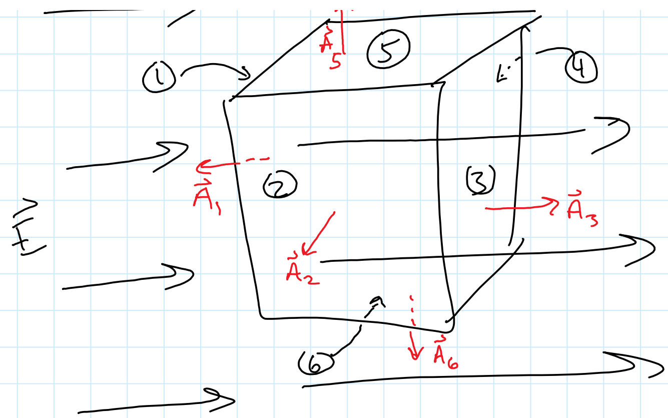
1) use $E = \int \frac{k\, dq}{r^2}$ integrate over charge distribution

2) use gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$



constant E field



constant
E field

Find Φ_E through each surface of the cube:

surface 1 : $\Phi = EA\cos 180^\circ = -EA$

2 $\Phi = EA\cos 90^\circ = 0$

3 $\Phi = EA\cos 0^\circ = EA$

4 $\Phi = EA\cos 90^\circ = 0$

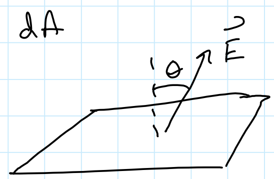
5 $\Phi = EA\cos 90^\circ = 0$

6 $\Phi = EA\cos 90^\circ = 0$

$\Phi_{net} = 0$

$$\Phi_E = \oint E \cdot dA = 0$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A} = \int E \cos\theta \, dA$$

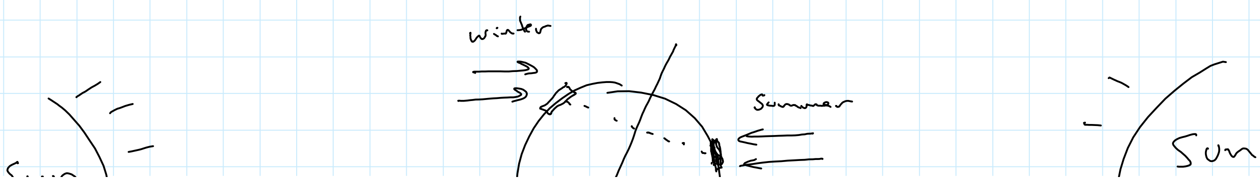


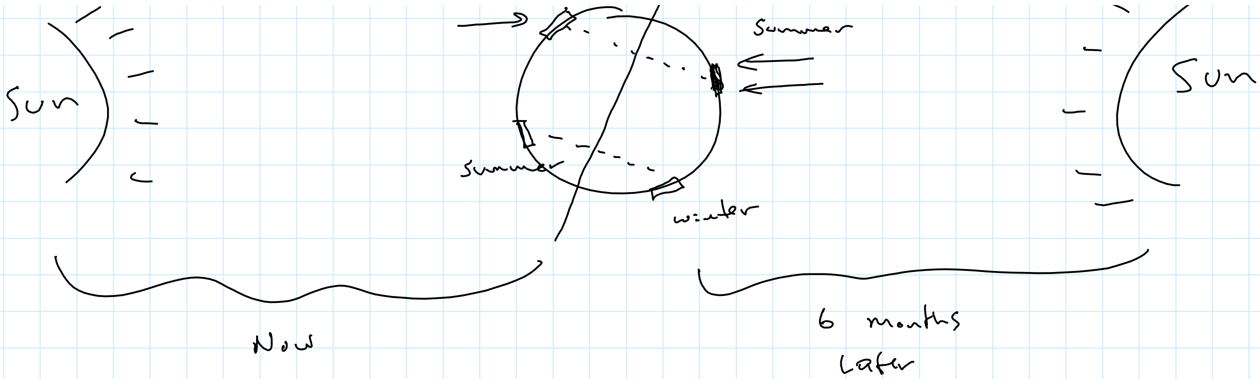
if E is constant over A:

$$\Phi = E\cos\theta \int dA$$

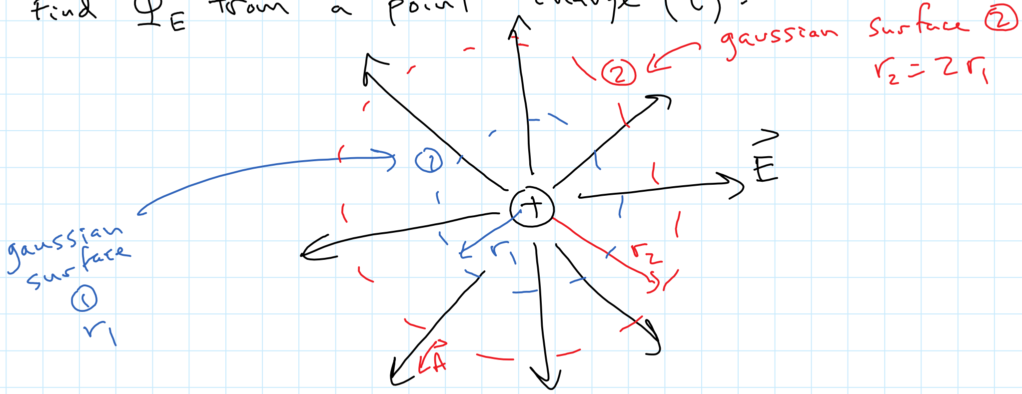
$$= EA\cos\theta$$

Aside: Seasons on Earth





Find Φ_E from a point charge (q):



gaussing surface ①

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$
$$= \oint E \, dA \cos 0^\circ$$
$$= E \int dA$$
$$= EA$$
$$= \left(\frac{Kq}{r_1^2}\right)\left(4\pi r_1^2\right)$$
$$= 4\pi K \, q$$
$$= \frac{1}{\epsilon_0} q$$
$$= \frac{q}{\epsilon_0}$$

gaussian surface ②

$$\Phi = \int \vec{E} \cdot d\vec{A}$$
$$= E \int dA$$
$$= EA$$
$$= \left(\frac{kq}{\cancel{r_2^2}}\right)\left(4\pi \cancel{r_2^2}\right)$$
$$= \frac{q}{\epsilon_0}$$

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worksheet e) $A_{II} = 4 \, A_I$ $A_{III} = 9 \, A_I$

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e) $A_{II} = 4\ A_{I}$ $\quad A_{III} = 9\ A_{I}$

f) $E_{II} = \frac{1}{4}\ E_{I}$ $\quad E_{III} = \frac{1}{9}\ E_{I}$

g) $\Phi_{II} = \Phi_{I}$ $\quad \Phi_{III} = \Phi_{I}$

same $\quad$ same

Total electric flux into or out of an enclosed surface is proportional to the <u>Net Charge</u> inside and does Not depend on surface Area

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h) $\Phi_{II} < \Phi_{I}$

$\Phi_{II} = 0$

J) same (# of lines is proportional to q)

k) q_1 is $+$ $\quad$ q_2 is $-$

L) $|\Phi_{I}| = |\Phi_{II}|$

m) Φ_{I} is $+$ $\quad$ Φ_{II} is $-$

n) $\Phi_{I} = -\Phi_{II}$

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Top:

$$\Phi_{I} = \frac{q_{in}}{\epsilon_0} = \frac{+q}{\epsilon_0}$$

$$\Phi_{II} = -\frac{q}{\epsilon_0}$$

$$\Phi_{III} = 0$$

$$\Phi_{IV} = \frac{q-q}{\epsilon_0} = 0$$

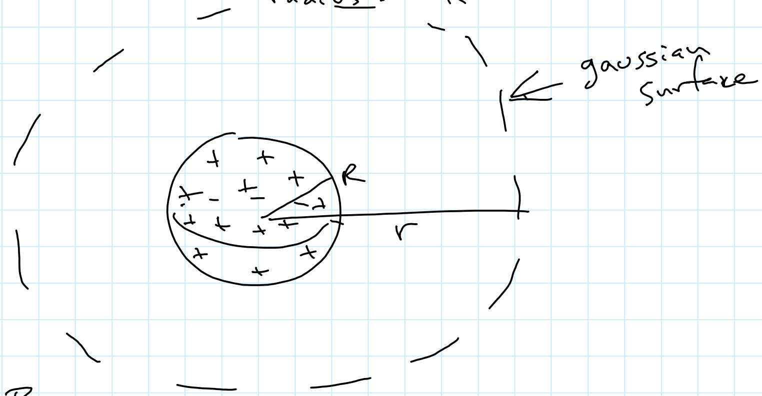
Find $\vec{E}$ using Gauss's Law:

i) Spherical symmetry

2) Cylindrical symmetry

3) infinite lines of charge or sheets of charge

Example: Find $\vec{E}$ everywhere due to a spherical charge distribution
uniform throughout volume
total charge : Q
radius : R



Outside
$r > R$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$\int \underbrace{E}_{\text{constant}} \underbrace{\cos\theta}_{1} \, dA = \frac{q_{in}}{\epsilon_0}$$

$$E \int dA = \frac{q_{in}}{\epsilon_0}$$

$$E A = \frac{q_{in}}{\epsilon_0}$$

$$E \, 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

$$= k \frac{Q}{r^2}$$

$k = \frac{1}{4\pi\epsilon_0}$

same as a pt charge

Inside of sphere
$r < R$

$$\int \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$E A = \frac{q_{in}}{\epsilon_0}$$

$$E \, 4\pi r^2 = q_{in}$$

$$E A = \frac{q_{in}}{\epsilon_0}$$

$$E\, 4\pi r^2 = \frac{q_{in}}{\epsilon_0}$$

find q_{in}:

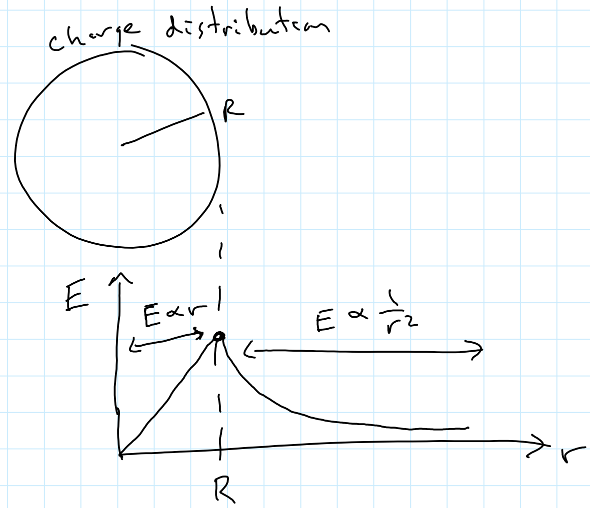
$$q_{in} = \frac{V_{in}}{V_{total}} Q = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} Q = \frac{r^3}{R^3} Q$$

$$E\, 4\pi r^2 = \frac{r^3}{R^3}\frac{Q}{\epsilon_0}$$

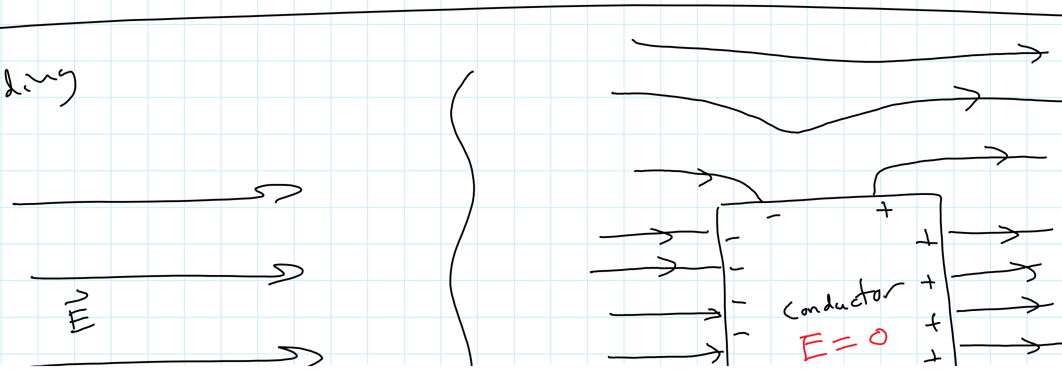
$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^3} r$$

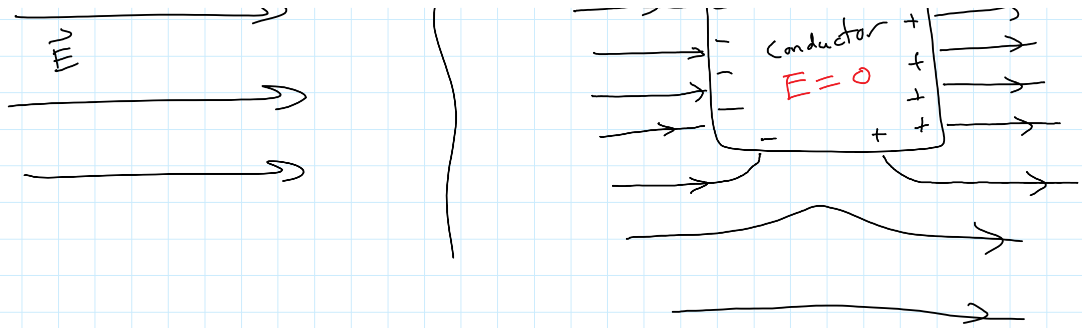
$$\vec{E} = \frac{kQ}{R^3} r$$ radially outward or $\hat{r}$

Plot E

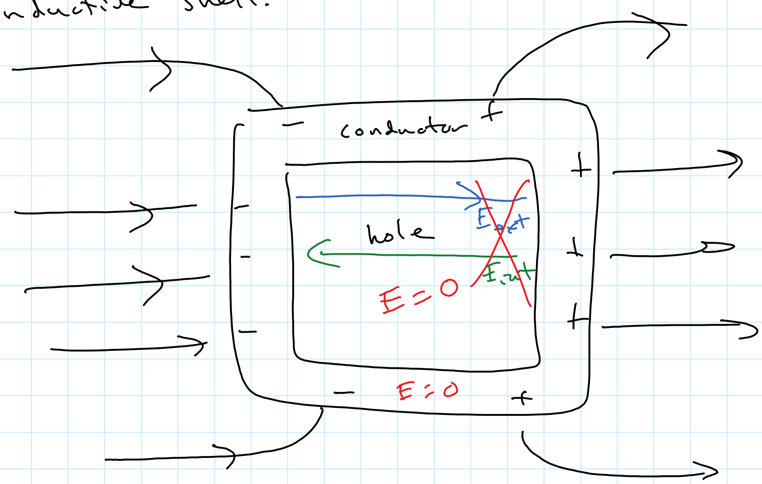


Shielding





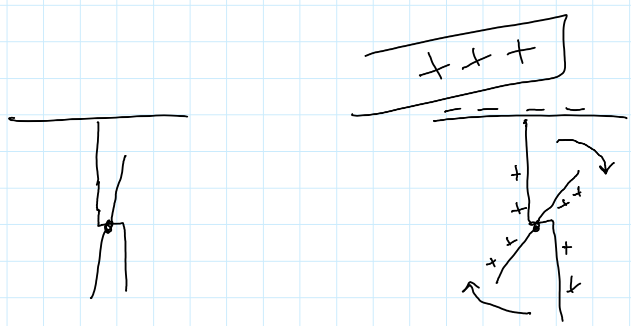
Conductive shell:



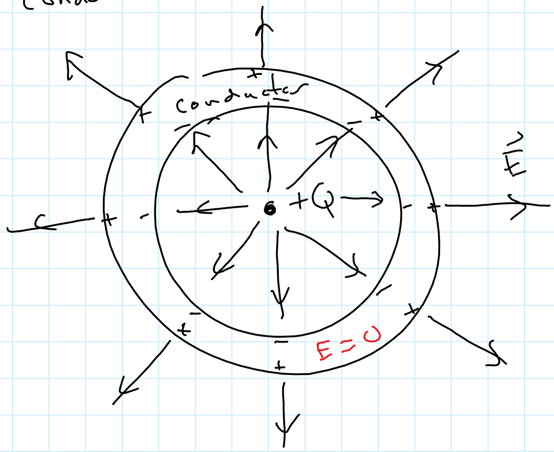
cavity is shielded by conductive shell

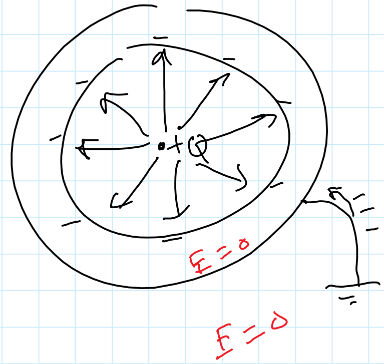
E = 0 inside cavity

must have a charge inside to have an E field inside the cavity of a conductive shell



conductive shell



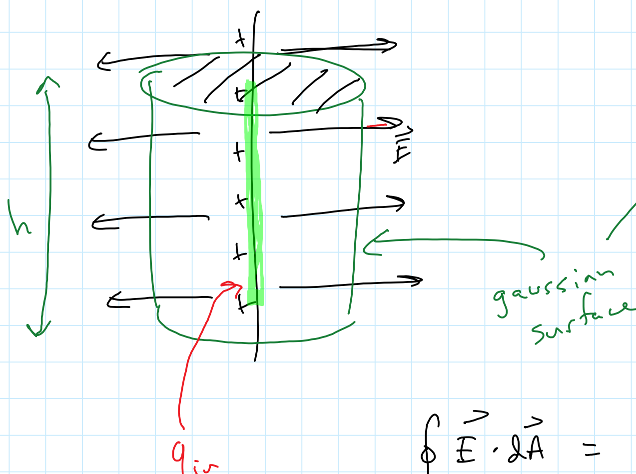


must be grounded to shield outside from E field on inside

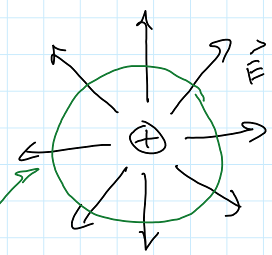
Example : Cylindrical Symmetry

long line of charge : λ

side view



top view



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$\underbrace{\int_{top} \vec{E}\cdot d\vec{A}}_{0} + \int_{side} \vec{E}\cdot d\vec{A} + \underbrace{\int_{bottom} \vec{E}\cdot d\vec{A}}_{0} = \frac{q_{in}}{\epsilon_0}$$

$$\int_{side} \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$E \int dA = \frac{q_{in}}{\epsilon_0}$$

$$E\, A_{side} = \frac{q_{in}}{\epsilon_0}$$

$$E\, 2\pi r h = \frac{q_{in}}{\epsilon_0}$$

$$E = \frac{1}{2\pi\epsilon_0} \underbrace{\frac{q_{in}}{h}}_{\lambda} \frac{1}{r}$$

$$\vec{E} = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r} \hat{r}$$