Phy 4B 4/13

Thursday, April 13, 2017 2:30 PM

Goals for the Lecture:

1) Understand electric fields and how charged objects behave in them
2) Be able to calculate electric fields from point charges
3) Be able to calculate electric fields from continuous charge distributions (1 – D only)
4) Understand electric field properties and how they are represented by electric field lines

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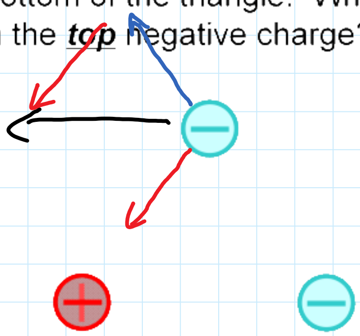
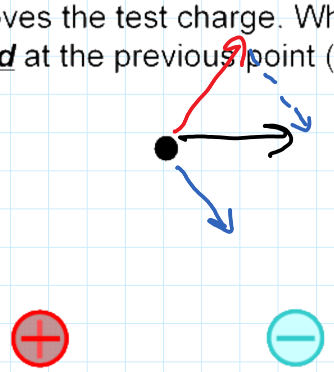
Tom places a negative charge at the top corner of an isosceles triangle to test the electric field produced by the +Q and –Q charges at the bottom of the triangle. What is the direction of the ***net force*** on the ***top*** negative charge?



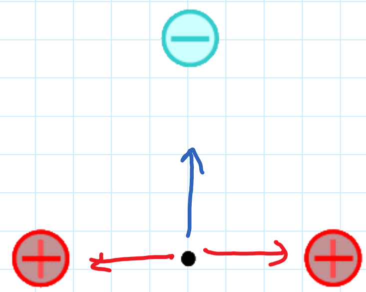
1. Left.
2. Down.
3. Right.
4. Up.
5. The net force is zero

Now, Tom removes the test charge. What is the direction of the ***electric field*** at the previous point (top of triangle)?



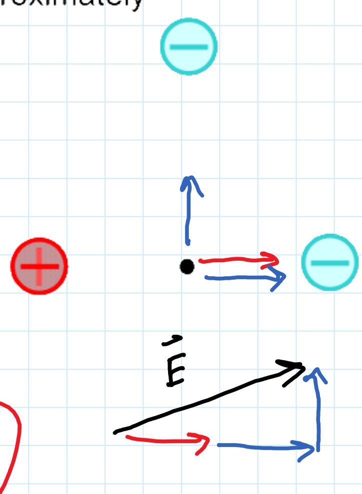
1. Left.
2. Down.
3. Right.
4. Up.
5. The electric field is zero

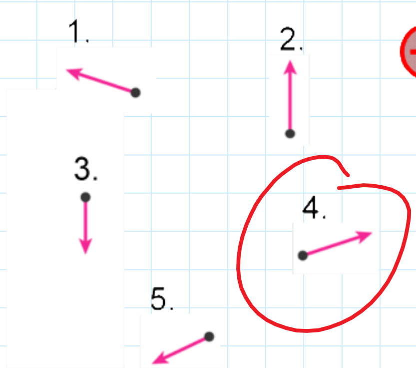
Tom never quits. He now wishes to find the direction of the electric field at the origin, as shown by the black dot. The ***electric field*** at the origin points



1. Left.
2. Down.
3. Right.
4. Up.
5. The net field is zero

Now, Tom changes one of the positive charges on the bottom to negative, as shown below. At the position of the dot, the ***electric field*** points approximately



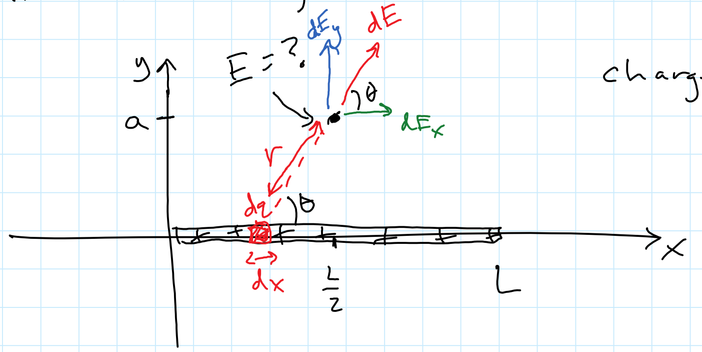


From multiple point charges:

$$\vec{E} = \sum \frac{k q_i}{r_i^2} \hat{r}_i \qquad \text{vector addition}$$

Continuous charge distribution:

Continuous charge distribution:



charged rod : length L
total charge +Q
uniformly distributed

$$dE = \frac{K\,dq}{r^2}$$

use components

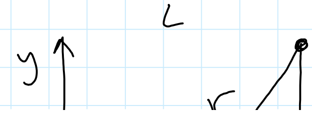
$$dE_x = dE\cos\theta = \frac{K\,dq}{r^2}\cos\theta \qquad dE_y = dE\sin\theta = \frac{k\,dq}{r^2}\sin\theta$$

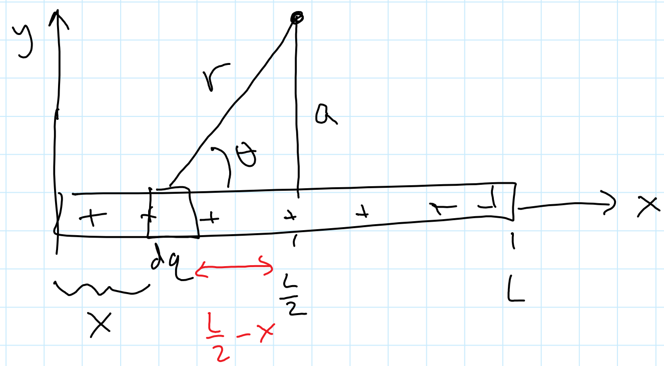
For our class, only 1-D charge Distributions

	charge density	units	dq
1-D charge distribution	$\lambda = \frac{Q}{L}$	$\frac{C}{m}$	$\lambda\,ds$
2-D	$\sigma = \frac{Q}{A}$	$\frac{C}{m^2}$	$\sigma\,dA$
3-D	$\rho = \frac{Q}{V}$	$\frac{C}{m^3}$	$\rho\,dV$

$$dE_x = \frac{K\,dq}{r^2}\cos\theta \qquad dE_y = \frac{K\,dq}{r^2}\sin\theta$$

$$dq = \lambda\,dx = \frac{Q}{L}\,dx$$





$$r^2 = a^2 + \left(\tfrac{L}{2} - x\right)^2$$

$$\cos\theta = \frac{\frac{L}{2} - x}{r}$$

$$\sin\theta = \frac{a}{r}$$

$$E_x = \int dE_x$$

$$= \int \frac{k\,dq}{r^2}\cos\theta$$

$$= \int \frac{k\,(\lambda\,dx)}{\underbrace{a^2 + \left(\frac{L}{2}-x\right)^2}_{r^2}} \; \frac{\frac{L}{2}-x}{\underbrace{\sqrt{a^2+\left(\frac{L}{2}-x\right)^2}}_{r}}$$

$$= \int_0^L \frac{k\,\lambda\,dx\,\left(\frac{L}{2}-x\right)}{\left[a^2 + \left(\frac{L}{2}-x\right)^2\right]^{3/2}}$$

integrate over charge distribution

$$= 0 \quad \text{by symmetry}$$

$$E_y = \int dE_y$$

$$= \int \frac{k\,dq}{r^2}\sin\theta$$

$$= \int \frac{k\,(\lambda\,dx)}{a^2 + \left(\frac{L}{2}-x\right)^2} \; \frac{a}{\sqrt{a^2+\left(\frac{L}{2}-x\right)^2}}$$

$$= \int_0^L \frac{k\,\lambda\,a\,dx}{\left[a^2 + \left(\frac{L}{2}-x\right)^2\right]^{3/2}}$$

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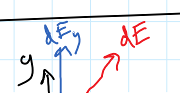
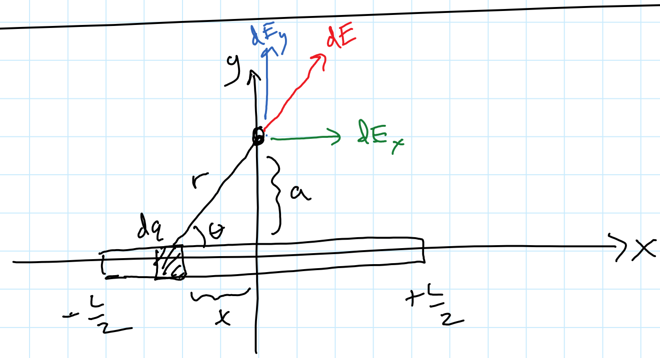
the



Change the origin



$$dE = \frac{k\ dq}{r^2}$$

$$dE_x = dE \cos\theta$$

$$= \frac{k\ dq}{r^2} \cos\theta$$

$$= \frac{k(\lambda\ dx)}{x^2+a^2} \underbrace{\frac{x}{\sqrt{x^2+a^2}}}_{\cos\theta}$$

$$dE_y = dE \sin\theta$$

$$= \frac{k\ dq}{r^2} \sin\theta$$

$$= \frac{k\ (\lambda\ dx)}{x^2+a^2} \underbrace{\frac{a}{\sqrt{x^2+a^2}}}_{\sin\theta}$$

$$E_x = \int_{-\frac{L}{2}}^{+\frac{L}{2}} \frac{k\lambda\ x\ dx}{(x^2+a^2)^{3/2}}$$

$$E_y = \int_{-\frac{L}{2}}^{+\frac{L}{2}} \frac{k\lambda a\ dx}{(x^2+a^2)^{3/2}}$$

$$= k\lambda a \left(\frac{1}{a^2} \frac{x}{\sqrt{x^2+a^2}}\right)\Bigg|_{-\frac{L}{2}}^{+\frac{L}{2}}$$

$$= \frac{k\lambda a}{a^2}\left(\frac{\frac{L}{2}}{\sqrt{\frac{L^2}{4}+a^2}} + \frac{\frac{L}{2}}{\sqrt{\frac{L^2}{4}+a^2}}\right)$$

$$= \frac{2k\lambda}{a} \frac{\frac{L}{2}}{\sqrt{\frac{L^2}{4}+a^2}} \quad \hat{y}$$

Worksheet p. 14

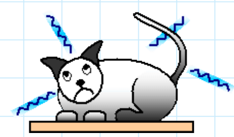
Top:

	True	False	Can't Determine
1)			X
2)	X		
3)			X
4)			X
5)		X	
6)			X

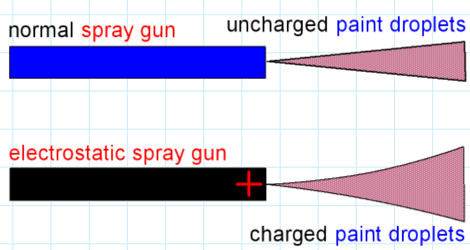
$$F_{AC} = \frac{k\, q_A q_C}{r_{AC}^2}$$

$$F_{BC} = \frac{k\, q_B q_C}{r_{BC}^2}$$

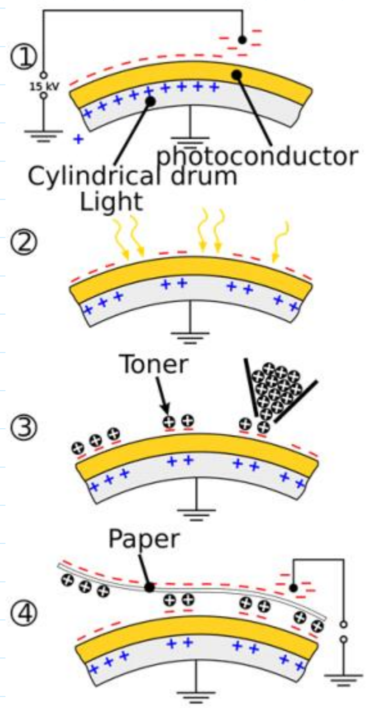
Electrostatics and everyday life



1. When you take clothes out of the drier and they cling together
2. Plastic Wrap (cling film) sticking to everything
3. Getting a small electric shock from a cat/dog that has rolled on a synthetic carpet
4. In a thunder storm there are huge flashes of lightning
5. An electrostatic dust collector in a chimney.
6. Paint sprays can be charged and the object they are spraying earthed to attract the paint towards it.



7. Photocopiers use a charged sheet to attract fine carbon powder



Worksheet
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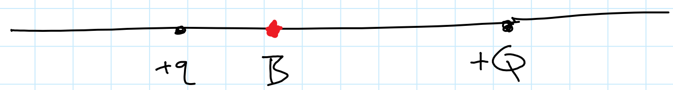
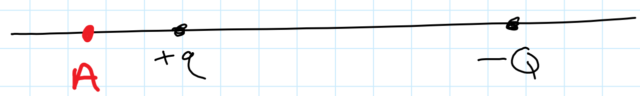
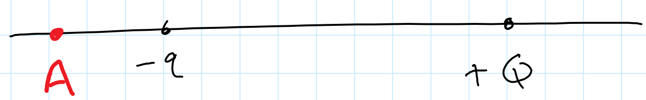
1)



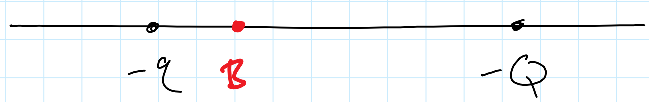
2)



3)



4)



Worksheet
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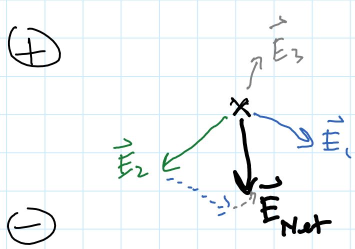


+

A ↓

B ↓

C ↑

D ↑

E ↑

F ↓

G ↑

H ↓

P. 22

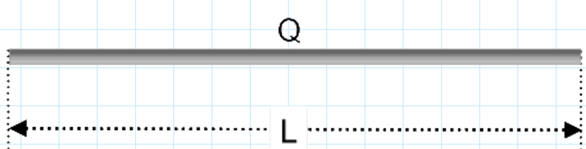
a) positive / Negative

b) tangent to field lines

c) more dense / closer together

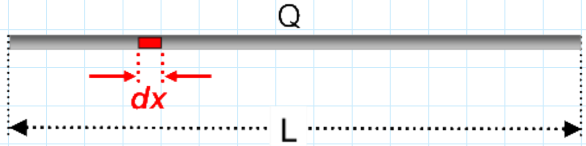
d)

A total charge Q is uniformly distributed over the length L of a line charge distribution. The charge density **λ** is given by



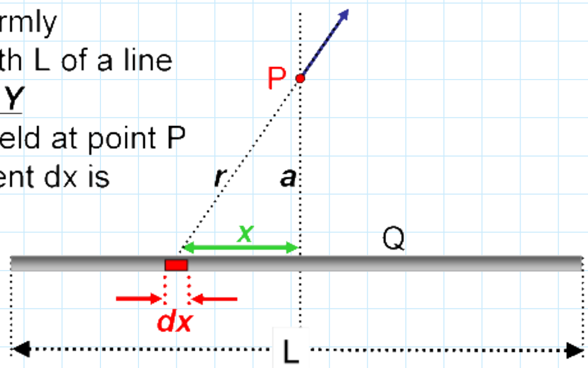
1. $\frac{Q}{L}$
2. $\left(\frac{Q}{L}\right)dx$
3. $\frac{L}{Q}$
4. Q
5. None of the above.

A total charge Q is uniformly distributed over the length L of a line charge distribution. The total charge inside a short element dx is given by



1. $\frac{Q}{L}$
2. $\left(\frac{Q}{L}\right)dx$
3. $\frac{L}{Q}$
4. Qdx
5. None of the above.

A total charge Q is uniformly distributed over the length L of a line charge distribution. ***The Y component*** of electric field at point P created by a short element dx is given by:



1. $\frac{K\frac{Q}{L}dx}{r^2} \times \frac{a}{r}$
2. $\frac{K\frac{Q}{L}dx}{r^2} \times \frac{x}{r}$ ← 2 is the X-comp
3. $\frac{K\frac{Q}{L}dx}{r^2} \times \frac{a}{x}$
4. $\frac{K\frac{Q}{L}dx}{r^2} \times \frac{x}{a}$

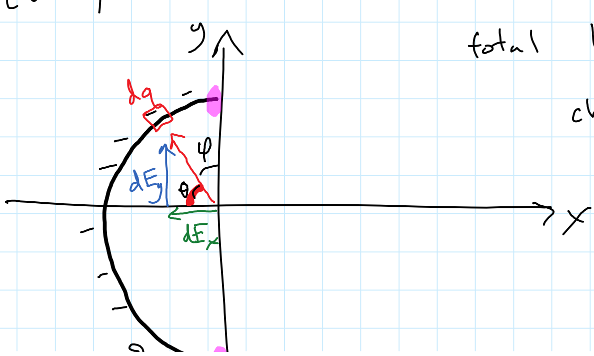
Find E at origin, due to semi-circle of charge

Curved line of charge

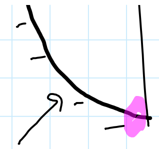
total charge: $Q = -7.5\ \mu C$

total length: $L = 14$ cm

charge is uniformly distributed



$r = \frac{0.14\text{ m}}{\pi}$



$$dE = \frac{K \, dq}{r^2}$$

using θ

$$dE_x = \frac{K \, dq}{r^2} \cos\theta \qquad\qquad dE_y = \frac{K \, dq}{r^2} \sin\theta$$

$$E_x = \int dE_x = \int \frac{K \, dq}{r^2} \cos\theta \qquad\qquad E_y = \int dE_y = \int \frac{K \, dq}{r^2} \sin\theta$$

r is constant

$$dq = \lambda \, ds = \lambda \underbrace{r \, d\theta}_{ds}$$

$$E_x = \int \frac{k \lambda r \, d\theta}{r^2} \cos\theta \qquad\qquad E_y = \int \frac{k \lambda r \, d\theta}{r^2} \sin\theta$$

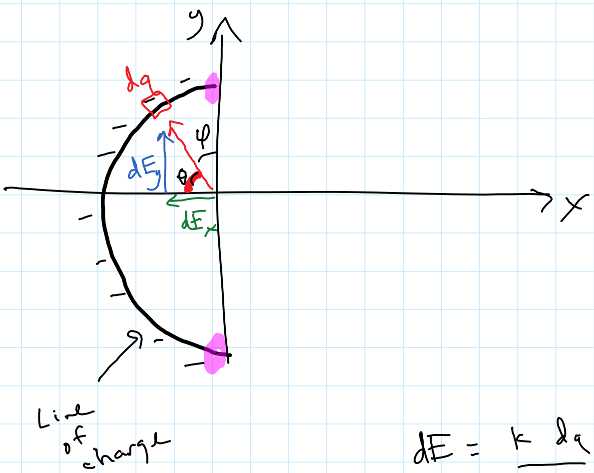
$$= \frac{k\lambda}{r} \int_{-\pi/2}^{+\pi/2} \cos\theta \, d\theta \qquad\qquad = \frac{k\lambda}{r} \int_{-\pi/2}^{+\pi/2} \sin\theta \, d\theta$$

$$= \frac{k\lambda}{r} \underbrace{\sin\theta \Big|_{-\pi/2}^{+\pi/2}}_{2} \qquad\qquad = \frac{k\lambda}{r} \underbrace{-\cos\theta \Big|_{-\pi/2}^{+\pi/2}}_{0}$$

$$= \frac{2k\lambda}{r} \left(-\hat{x}\right) \qquad\qquad = 0$$

$$= -2.16 \times 10^{7} \, \frac{N}{C} \, \hat{x}$$

above problem again, but using φ instead of θ



$$dE = \frac{k\,dq}{r^2}$$

using φ

$$dE_x = dE \sin\varphi$$
$$= \frac{k\,dq}{r^2}\sin\varphi$$

$$E_x = \int dE_x = \int_0^{\pi} \frac{k\lambda r\,d\varphi}{r^2}\sin\varphi$$
$$= \frac{k\lambda}{r}\int_0^{\pi}\sin\varphi\,d\varphi$$
$$= \frac{k\lambda}{r}\underbrace{\left(-\cos\varphi\Big|_0^{\pi}\right)}_{2}$$
$$= \frac{2k\lambda}{r}\,(-\hat{x})$$

$$dE_y = dE\cos\varphi$$
$$= \frac{k\,dq}{r^2}\cos\varphi$$

$$E_y = \int dE_y$$
$$= \frac{k\lambda}{r}\int_0^{\pi}\cos\varphi\,d\varphi$$
$$= \frac{k\lambda}{r}\sin\varphi\Big|_0^{\pi}$$
$$= 0$$

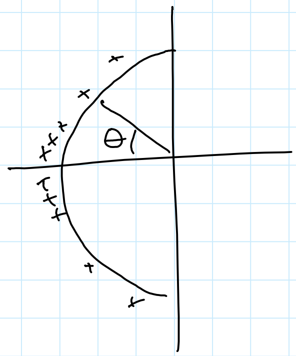
Non-uniform charge distribution:

same as before, but $\lambda = \alpha \cos\theta$

Non-uni...

Same as before, but $\lambda = \alpha \cos\theta$

$\uparrow$ positive constant



$$dE_x = \frac{k\,dq}{r^2}\cos\theta$$

$$= \frac{k(\lambda\, r\, d\theta)}{r^2}\cos\theta$$

$$= \frac{k(\alpha\cos\theta\, r\, d\theta)}{r^2}\cos\theta$$

$$E_x = \int dE_x = \frac{k\alpha}{r}\int_{-\pi/2}^{+\pi/2}\cos^2\theta\, d\theta$$