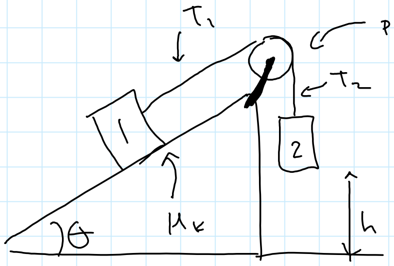
Phy 4A 11/23

Wednesday, November 23, 2016 9:56 AM

Goals for the Lecture:

1) Be able to use conservation of angular momentum to solve rotational collision problems

Example



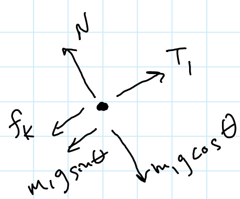
Pulley is a solid disk

given: $m_1, m_2, m_p, R_{pulley}$

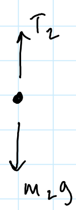
θ, μ_k

Find: T_1, T_2, a

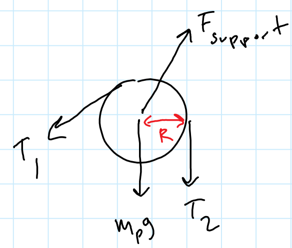
M_1



m_2



Pulley



$\Sigma F_1 = m_1 a$ $+\nearrow$

$T_1 - f_k - m_1 g\sin\theta = m_1 a$

$$\boxed{T_1 - \mu_k m_1 g\cos\theta - m_1 g\sin\theta = m_1 a}$$

$\Sigma F_2 = m_2 a$ $\downarrow +$

$$\boxed{m_2 g - T_2 = m_2 a}$$

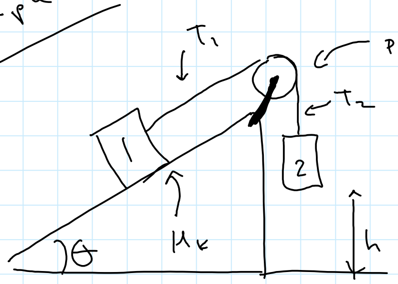
$\Sigma \vec{\tau} = I\alpha$ $+\circlearrowright$

$T_2 R - T_1 R + F_{support}(0) + m_p g(0) = I\alpha$

$T_2 R - T_1 R = \left(\frac{1}{2} m_p R^2\right)\left(\frac{a}{R}\right)$

$$\boxed{T_2 - T_1 = \frac{m_p}{2} a}$$

Example



Pulley is a solid disk

given: $m_1, m_2, m_p, R_{pulley}$

$\theta, \mu_k, h, v_i = 0$

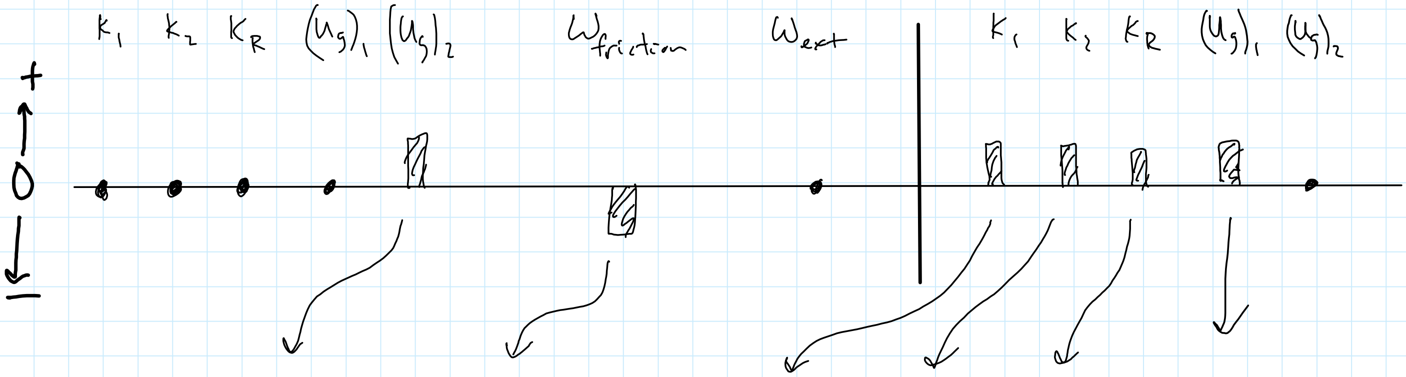
Find: find v_f (just before m_2 hits the ground)

Initial

During

Final

Initial E_i During Final E_f



$$m_2 g h - \mu_k m_1 g \cos\theta \, h = \frac{1}{2} m_1 V_f^2 + \frac{1}{2} m_2 V_f^2 + \frac{1}{2} I \omega_f^2 + m_1 g h \sin\theta$$

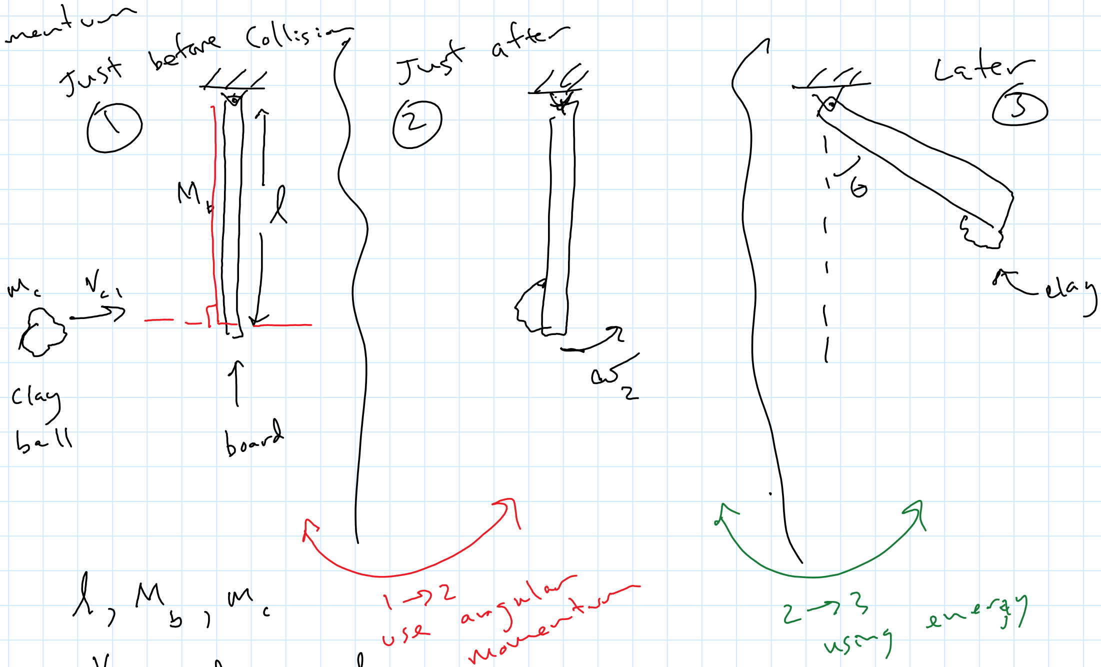
$I = \frac{1}{2} m_p R^2$, $\omega_f = \frac{V_f}{R}$

$$= \frac{1}{2} m_1 V_f^2 + \frac{1}{2} m_2 V_f^2 + \frac{1}{2}\left(\frac{1}{2} m_p \cancel{R^2}\right) \frac{V^2}{\cancel{R^2}} + m_1 g h \sin\theta$$

$$= \frac{1}{2}\left(m_1 + m_2 + \frac{m_p}{2}\right) V_f^2 + m_1 g h \sin\theta$$

Solve for V_f

Angular momentum



given: l, M_b, m_c

V_{c1}, and $r_c \ll l$

find: θ when the object comes to rest

$\underline{1 \to 2}$

$$L_1 = L_2 \qquad + \circlearrowleft$$

$$L_{clay} + L_{board} = L_{clay+board}$$

$$\underbrace{m_c v_{c_1} \ell}_{\text{object in straight Line}} + 0 = \underbrace{I_{total} \; \omega_2}_{\text{object rotating}}$$

$L = m v d_\perp$ for object moving in a straight Line

$L = I \omega$ for a rotating object

$$m_c v_{c_1} \ell = (I_c + I_b)\, \omega_2$$

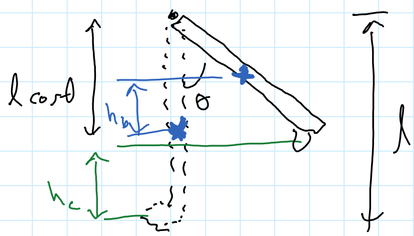
$$m_c v_{c_1} \ell = \left(m_c \ell^2 + \tfrac{1}{3} m_b \ell^2\right) \omega_2$$

solve for ω_2

$\underline{2 \to 3}$

$$(K_R)_2 = (U_g)_3$$

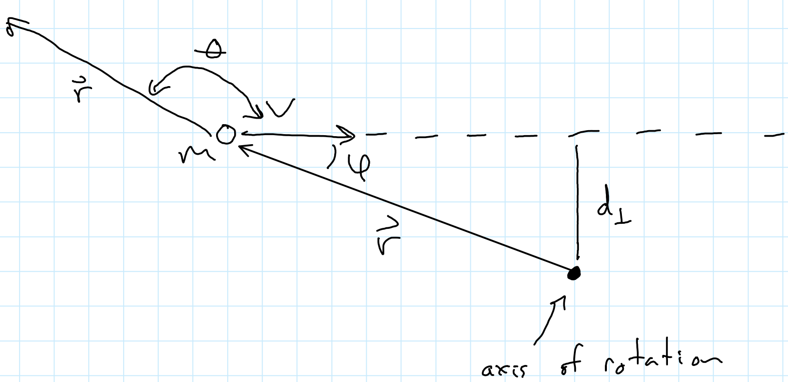
$$\tfrac{1}{2} I_{total}\, \omega_2^2 = m_c g h_c + m_b g h_b$$



$$h_c = \ell - \ell\cos\theta$$

$$h_b = \tfrac{1}{2}(\ell - \ell\cos\theta)$$

Angular momentum: object moving in a straight line



axis of ro

$$\vec{L} = \vec{r} \times \vec{p} \qquad \text{OR} \qquad L = p\, d_\perp$$

$$= |\vec{r}||\vec{p}| \sin\theta \qquad\qquad = mv\, d_\perp$$

$$= r\, mv \sin(180 - \varphi)$$

$$= r\, mv \sin\varphi$$

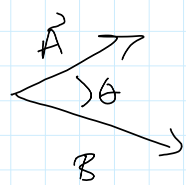
$$= mv \underbrace{r \sin\varphi}_{d_\perp}$$

$$= mv\, d_\perp$$

Right hand rule:

$$\vec{C} = \vec{A} \times \vec{B}$$

$$|\vec{C}| = |\vec{A}|\,|\vec{B}| \sin\theta$$



direction from Rt hand rule

into page: $\otimes$

out of page: $\odot$

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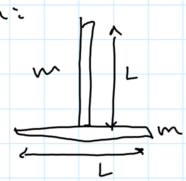
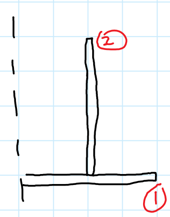
worksheet p. 106

Find I for each in terms of L and m

each arm:



A)



$$I = I_1 + I_2$$

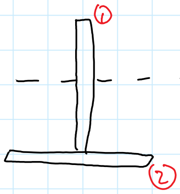
$$I_1 = \frac{1}{3} mL^2$$

$$I_2 = mr^2 = m\left(\frac{L}{2}\right)^2 = \frac{1}{4} mL^2$$

$$= \left(\frac{1}{3} + \frac{1}{4}\right) mL^2$$

$$= \frac{7}{12} mL^2$$

$\frac{7}{12} mL^2$

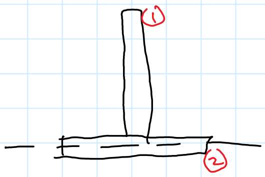
D) 

$I = I_1 + I_2$

$I_1 = \frac{1}{12} mL^2$

$I_2 = mr^2 = m\left(\frac{L}{2}\right)^2 = \frac{1}{4} mL^2$

$I = \left(\frac{1}{12} + \frac{1}{4}\right) mL^2 = \frac{1}{3} mL^2$

E) 

$I = I_1 + I_2$

$I_1 = \frac{1}{3} mL^2$

$I_2 = 0$

$I = \frac{1}{3} mL^2$

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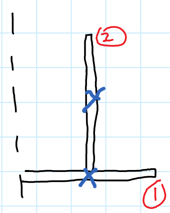
p. 107 top Find the torque from gravity for each

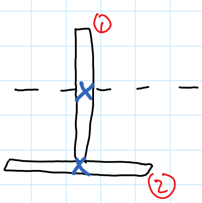
A) 

$\tau = \tau_1 + \tau_2$

$\tau_1 = mg\frac{L}{2}$

$\tau_2 = mg\frac{L}{2}$

$\tau = mgL$

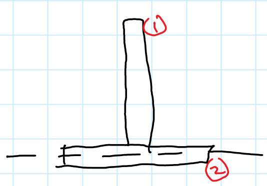
D) 

$\tau = \tau_1 + \tau_2$

$\tau_1 = 0$

$\tau_2 = mg\frac{L}{2}$

$\tau = \frac{1}{2} mgL$

E) 

$\tau = \tau_1 + \tau_2$

$= \frac{1}{2} mgL + 0$

$= \frac{1}{2} mgL$

p. 107 bottom:

$\Sigma \vec{\tau} = I \vec{\alpha}$

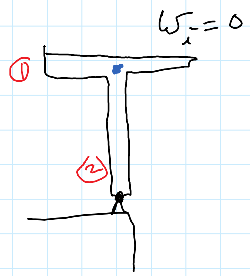
$\alpha = \frac{\Sigma \tau}{I}$

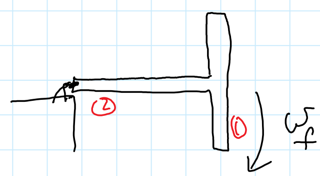
I

A) $\alpha = \dfrac{\cancel{m} g L}{\frac{7}{12}\cancel{m} L^2} = \dfrac{12\, g}{7 L}$

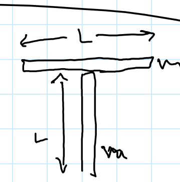
D) $\alpha = \dfrac{\frac{1}{2} mgL}{\frac{1}{3} mL^2} = \dfrac{3}{2}\,\dfrac{g}{L}$

E) $\alpha = \dfrac{\frac{1}{2} mgL}{\frac{1}{3} mL^2} = \dfrac{3}{2}\,\dfrac{g}{L}$





given:



find: ω_f

$$E_i = E_f$$

$$(U_g)_i = (K_R)_f$$

$$(U_g)_1 + (U_g)_2 = (K_R)_f$$

$$mgL + mg\frac{L}{2} = \frac{1}{2} I_{total}\, \omega_f^2$$

$$\frac{3}{2} mgL = \frac{1}{2}\left(I_1 + I_2\right)\omega_f^2$$

$$I_1 = I_{cm} + md^2$$

$$= \frac{1}{12} ML^2 + mL^2 = \frac{13}{12} mL^2$$

$$I_2 = \frac{1}{3} mL^2$$

$$I_{total} = \left(\frac{1}{3} + \frac{13}{12}\right) mL^2 = \frac{17}{12} ML^2$$

$$\frac{3}{\cancel{2}} \cancel{m} gL = \frac{1}{\cancel{2}}\left(\frac{17}{12} \cancel{m} L^2\right)\omega_f^2$$

$$\sqrt{\frac{36}{17}\ \frac{g}{L}} = \omega_f$$