Phy 4A 11/21

Monday, November 21, 2016 9:59 AM

**Goals for the Lecture:**

1) Know how to calculate the angular momentum of objects moving in a straight lines
2) Know how to calculate the angular momentum of rotating rigid objects

Parallel axis Theorem:

$$I = I_{cm} + MD^2$$

$I$: rotational inertia about an axis parallel to the c of m axis

$I_{cm}$: rotational inertia about an axis of rotation through the c of m

$M$: total mass of object

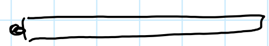
$D$: distance between c of m axis and parallel axis

Given:

Axis through c of m

$$I_{cm} = \frac{1}{12} ML^2$$

Find



using parallel axis theorem:

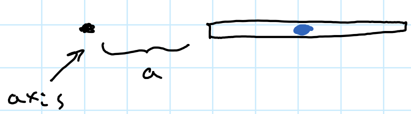
$$I = I_{cm} + MD^2$$

$$= \frac{1}{12} ML^2 + M\left(\frac{L}{2}\right)^2$$

$$= \left(\frac{1}{12} + \frac{1}{4}\right) ML^2$$

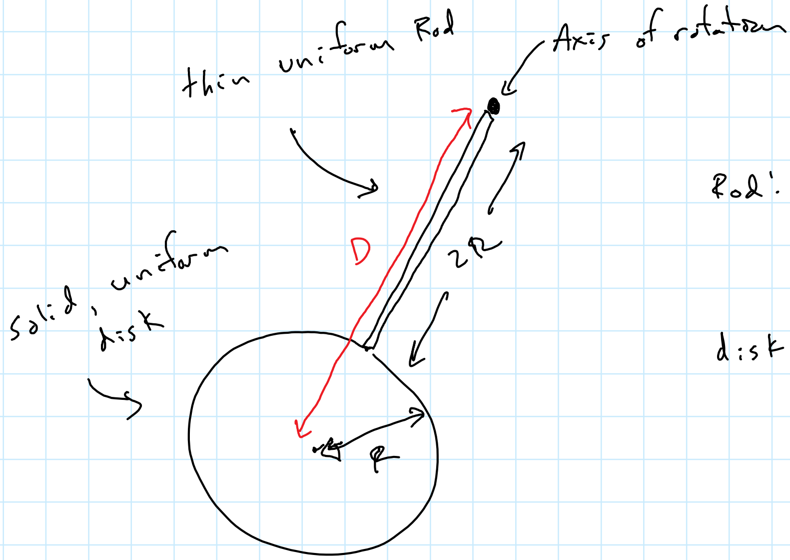
$$= \frac{1}{3} ML^2$$

Find:




$$I = \frac{1}{12} ML^2 + M\left(\frac{L}{2} + a\right)^2$$

Find $I$ for the pendulum of a grandfather clock:




Rod: length $= 2R$
mass $= M$

disk: radius $= R$
mass $= 2M$

$$I = I_{rod} + I_{disk}$$

$$I_{rod} = \frac{1}{3} M_{rod} L_{rod}^2 = \frac{1}{3} M (2R)^2 = \frac{4}{3} M R^2$$

$$I_{disk} = I_{cm} + M_{disk} D^2$$

$$= \frac{1}{2} M_{disk} R^2 + M_{disk} (3R)^2$$

$$= \frac{1}{2} (2M) R^2 + (2M)\, 9R^2$$

$$= MR^2 + 18\, MR^2$$

$$= 19\, MR^2$$

$$I = \frac{4}{3} MR^2 + 19\, MR^2$$

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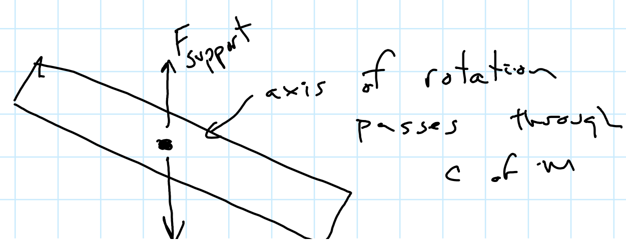
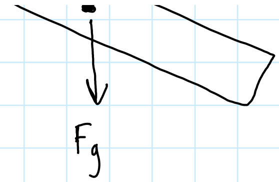





c of m

Equilibrium:

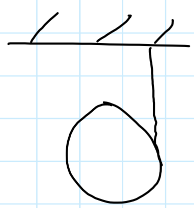
$\sum \vec{F} = 0$ : $\vec{F}_{support} + \vec{F}_g = 0$ $|\vec{F}_{support}| = |\vec{F}_g|$

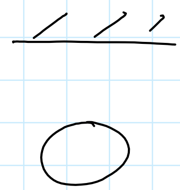
and

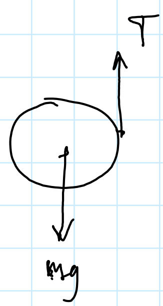
$\sum \vec{\tau} = 0$ $F_{support}(0) + F_g(0) = 0$

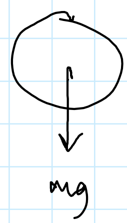
No torque

FBD








which hits first: B

Does A hit the "X" or off to one side: A hits the "X"

p. 94 B) $(F_{Net})_A = Mg - T = ma$

$$a = \frac{mg - T}{m} = g - \frac{T}{m}$$

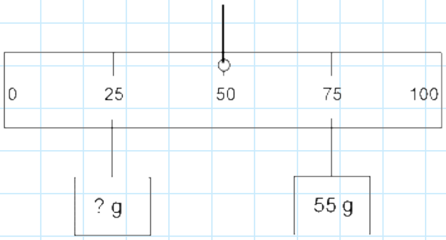
$(F_{Net})_B = mg = ma$

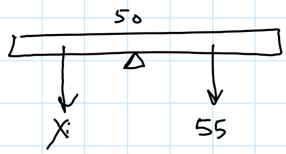
$$a = g$$

# Torque Example - 1

A 145 g meter stick is suspended at the 50 cm mark.
If 55 g are added at the 75 cm mark, how many grams should be added at the 25 cm mark to keep the system in equilibrium?

1) 25 g
2) 55 g
3) 75 g
4) 145 g
5) 1495 g




$$\sum \vec{\tau} = 0$$

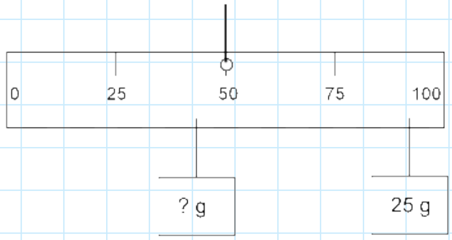
$$+X(\cancel{25}\,\text{cm}) - 55(\cancel{25}\,\text{cm}) = 0$$

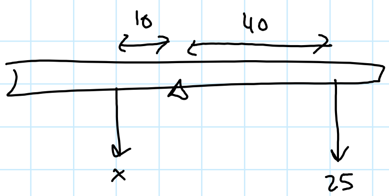
$$X = 55$$

# Torque Example - 2

A 145 g meter stick is suspended at the 50 cm mark.
If 25 g are added at the 90 cm mark, how many grams should be added at the 40 cm mark to keep the system in equilibrium?

1) 25 g
2) 40 g
3) 90 g
4) 100 g
5) 1495 g



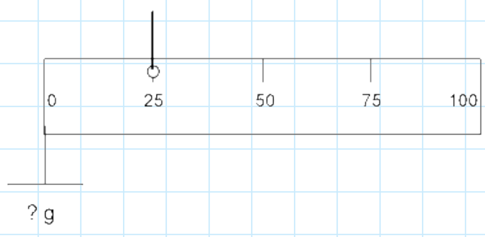



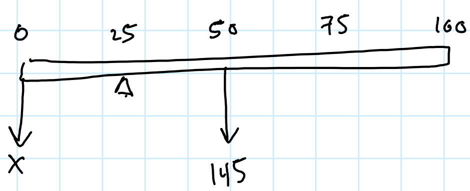

$$\Sigma \tau = x(10) - 25(40) = 0 \Rightarrow x = 100$$

# Torque Example - 3

A 145 g meter stick is suspended at the 25 cm mark.
How many grams should be added at the zero cm mark to keep the system in equilibrium?

1) 25 g
2) 55 g
3) 90 g
4) 100 g
5) 145 g
6) 1495 g



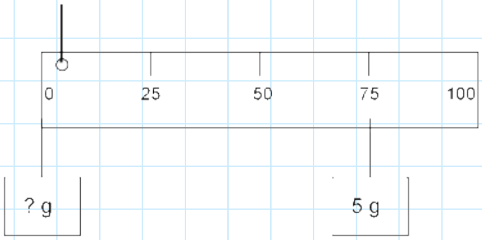



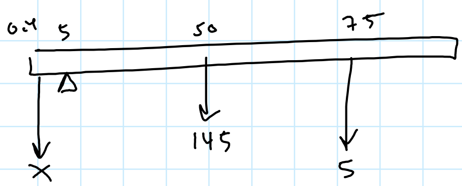

$$\Sigma \tau = x(25) - 145(25) = 0 \Rightarrow x = 145$$

# Torque Example - 4

A 145 g meter stick is suspended at the 5 cm mark.
If 5 g are added at the 75 cm mark, how many grams should be added at the 0.4 cm mark to keep the system in equilibrium?

1) 25 g
2) 55 g
3) 90 g
4) 100 g
5) 145 g
6) 1495 g

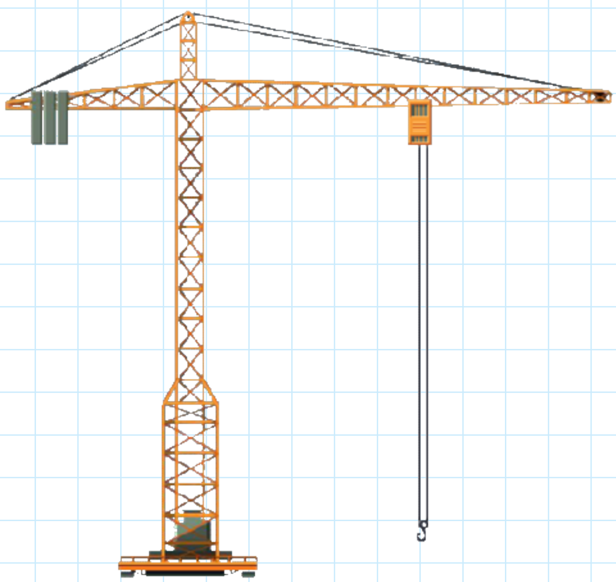


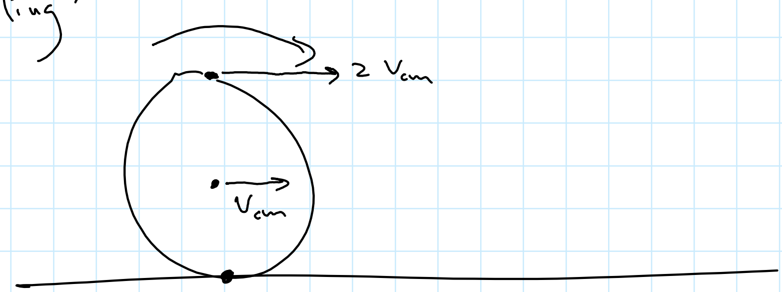

$$\Sigma \tau = x(4.6) - 145(45) - 5(70) = 0$$

$$x = 1495 \text{ grams}$$

# Torque Application

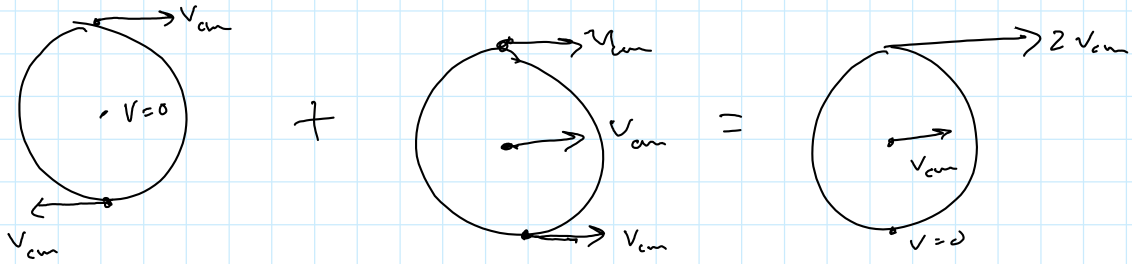


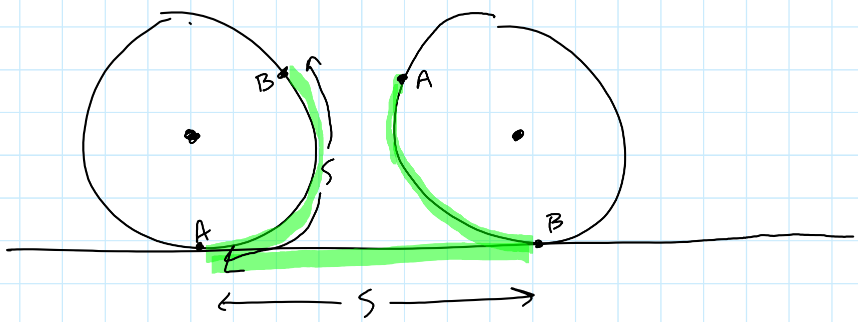
Rolling:



$v = 0$

Pure rotation + Pure Translation = Rolling




$$V_{rim} = r\omega$$

$$V_{cm} = V_{rim}$$

$$V_{cm} = r\omega \quad \text{rolling without slipping}$$

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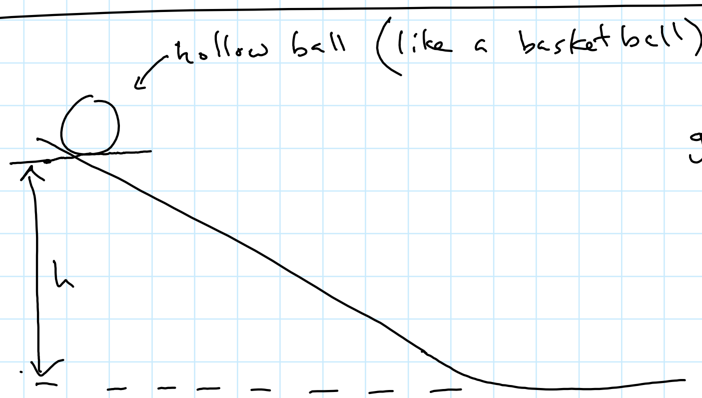


ball rolls without slipping

given: mass of ball $= m$

radius $= R$

$V_i = 0$

Starting height $= h$

find: $V_{cm}$ at bottom

$$E_i = E_f \quad \text{No rolling friction}$$

Rolling friction

bunches up
deforms

causes loss of mechanical energy

$$mgh = K_T + K_R$$

$$= \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2$$

about c of m

$$= \frac{1}{2} m v_{cm}^2 + \frac{1}{2} \left(\frac{2}{3} MR^2\right)\left(\frac{v_{cm}}{R}\right)^2$$

$v_{cm} = R\omega$

$$= \frac{1}{2} m v_{cm}^2 + \frac{1}{3} m v_{cm}^2$$

$$= \frac{5}{6} m v_{cm}^2$$

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Angular Momentum:

$$\sum \vec{F} = m\vec{a}$$
$$= m \frac{d\vec{v}}{dt}$$
$$= \frac{d(m\vec{v})}{dt}$$

$\vec{p} = m\vec{v}$

$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{r} \times \left(\sum \vec{F}\right) = \vec{r} \times \left(\frac{d\vec{p}}{dt}\right)$$

$$\sum \vec{\tau} = \frac{d}{dt}(\vec{r} \times \vec{p})$$

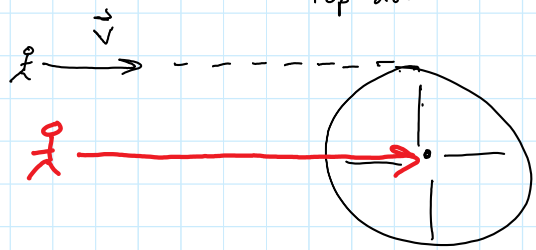
$\vec{l} = \vec{r} \times \vec{p}$ For objects or

moving in straight Lines

$\vec{L} = I\vec{\omega}$ For rotating objects

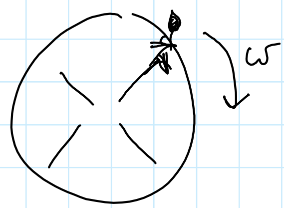
Top down view

initial




kid jumps on rotating disk

final




$P_i \neq P_f$ disk is attached to earth

$L_i = L_f$ No torque