Phy 4A 11/16

Wednesday, November 16, 2016 9:58 AM

Goals for the Lecture:

1) Understand how to use rotational kinematics equations to solve rotation problems
2) Understand what torque is and how to calculate it

Goals for the Lecture:

3) Understand how to use Newton's 2nd Law for rotation ($\sum \tau = I\alpha$) to solve problems
4) Know how to use the chart of rotational inertias to find rotational inertia of common shapes about typical axes of rotation
5) Know how to use the Parallel Axis Theorem, in conjunction with the chart of rotational inertias, to find the rotational inertia about any parallel axis of rotation
6) Know how to integrate to find the rotational inertia of one dimensional objects, including objects with non-uniform mass density

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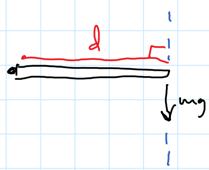
Top: Find Torque for each

E)



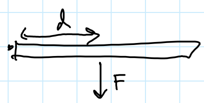
$\tau = F d$

$d = 1\,m$

$F = (1\,kg)\left(10\,\frac{m}{s^2}\right)$

$\tau = 10\,Nm$

B)

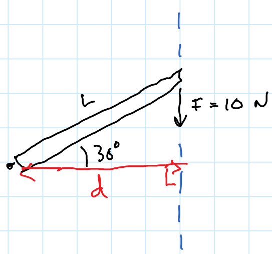


$d = 0.5\,m$

$F = 10\,N$

$\tau = 5\,Nm$

A)



$\tau = F d$

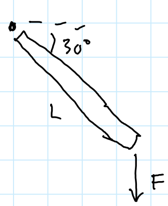
$F = 10\,N$

$d = L \cos 30°$

$= 0.87$

$\tau = 8.7\,Nm$

C)



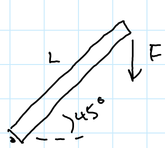
$\tau = F d$

$F = 10\,N$

$d = L \cos 30° = 0.87$

$\tau = 8.7\,Nm$

D)



$\tau = F d$

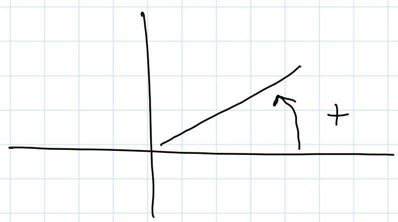
$F = 10\,N$

$d = L \cos 45° = 0.707$

$\tau = 7.1\,Nm$

$E > A = C > D > B$

Bottom: $\tau_A > \tau_B$



use Right hand rule for ω

$\vec{\omega}$ and $\vec{\alpha}$ point along axis of rotation

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$\Sigma \vec{F} = m\vec{a}$ Newton's 2nd Law

$\tau = rF$ if $F \perp r$

$a = r\alpha$

$$\tau = rF$$
$$= r(ma)$$
$$= r\,m\,(r\alpha)$$

$$\tau = mr^2\alpha$$

$$\Sigma\tau = \left(\Sigma mr^2\right)\alpha$$
$$\Sigma F = m\,a$$

inertia

Rotational inertia:

1) For a point object: $I = mr^2$

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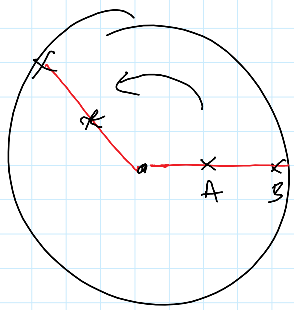
2) For multiple pt objects: $I = \sum m_i r_i^2$

3) For solid objects : $I = \int r^2 \, dm$

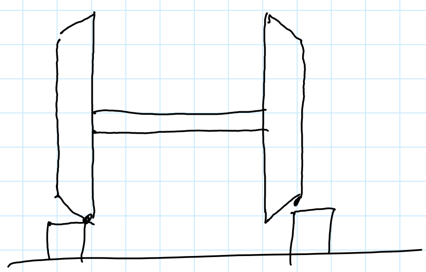


rotational velocity: $\omega_A = \omega_B$

linear velocity: $V_B > V_A$

$$V = r\omega$$

Train wheels



self correcting

always goes to center of tracks

$V = r\omega$

Worksheet
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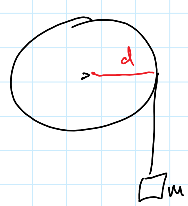
Top: A)



$\tau = Fd$

$F = (0.5 \text{ kg})(10 \frac{\text{m}}{\text{s}^2})$

$= 5 \text{ N}$

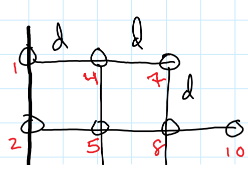
$d = (\text{radius})$

$= 0.1 \text{ m}$

$\tau = 0.5 \text{ Nm}$

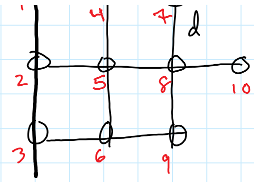
bottom: Find I for each (in terms of d and m)

A)



each has mass m

spacing $= d$



spacing = d

$$I = \sum m_i r_i^2$$

$$= m_1 r_1^2 + m_2 r_2^2 + \cdots + m_{10} r_{10}^2$$

$$= \underbrace{m(0)}_{m_1} + \underbrace{m(0)}_{2} + \underbrace{m(0)}_{3} + \underbrace{m(d)^2}_{4} + \underbrace{m(d)^2}_{5} + \underbrace{m(d)^2}_{6} + \underbrace{m(2d)^2}_{7} + \underbrace{m(2d^2)}_{8} + \underbrace{m(2d^2)}_{9} + \underbrace{m(3d)^2}_{10}$$

$$= 3(md^2) + 3(m4d^2) + m9d^2$$

$$= 24\, md^2$$

B) $I = 2[m(2d)^2] + 3[md^2] + 3(0) + 2(md^2)$

$$= 8md^2 + 3md^2 + 0 + 2md^2$$

$$= 13\, md^2$$

C) $I = 3(0) + 2(md^2) + 2[m(2d)^2] + 3[m(3d)^2]$

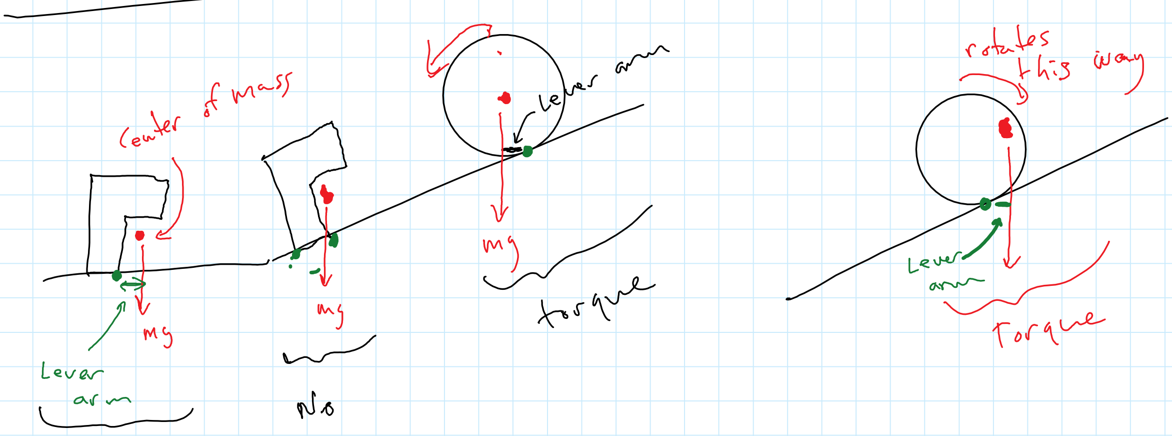
$$= 2md^2 + 8md^2 + 27md^2$$

$$= 37\, md^2$$

D) $I = 2[md^2] + 3(0) + 2(md^2) + 3[m(2d)^2]$

$$= 2md^2 + 2md^2 + 12md^2$$

$$= 16\, md^2$$



$\underbrace{\text{Lever arm}}$

Torque

No Torque

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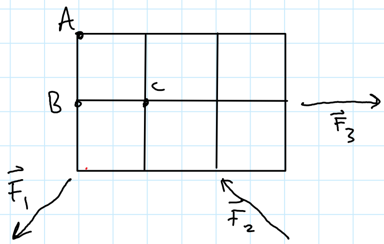

a) clockwise (CW)

b) CW

c) zero

d) zero

e) CCW

f) zero

g) CCW

h) zero

i) zero

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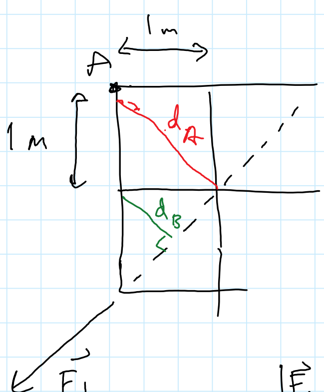
	A	B	C
Torque from F_1 about  $\lvert\vec{F}_1\rvert = \lvert\vec{F}_2\rvert = \lvert\vec{F}_3\rvert = F$	$F_1 d_A = F\sqrt{2}\,(1m) = F\sqrt{2}$	$F_1 d_B = F\frac{\sqrt{2}}{2}$	0
Torque from F_2 about	0	$F\frac{\sqrt{2}}{2}$	0

Torque from F_2 about	0	$F\frac{\sqrt{2}}{2}$	0
Torque from F_3 about	$F(1\,m) = F$	0	0

Net torque:

A: $\vec{\tau}_A = \vec{\tau}_{F_1} + \vec{\tau}_{F_2} + \vec{\tau}_{F_3}$ $\quad +\circlearrowleft$

$= -F\sqrt{2} + 0 + F$

$= (1-\sqrt{2})F$ $\quad$ Negative, so it rotates $\curvearrowright$ CW

B: $\vec{\tau}_B = \vec{\tau}_{F_1} + \vec{\tau}_{F_2} + \vec{\tau}_{F_3}$ $\quad +\circlearrowleft$

$= -F\frac{\sqrt{2}}{2} + F\frac{\sqrt{2}}{2} + 0$

$= 0$ $\quad$ No torque

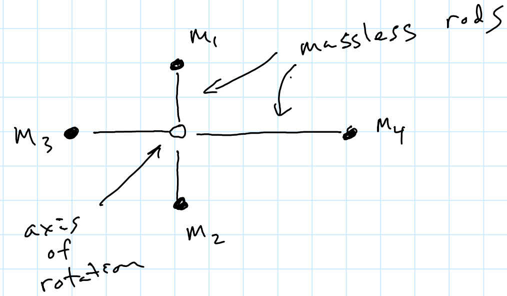
No rotation (if initially at rest)

C: $\vec{\tau}_c = \vec{\tau}_{F_1} + \vec{\tau}_{F_2} + \vec{\tau}_{F_3}$

$= 0 + 0 + 0$

$= 0$

Kinetic energy:



What is the kinetic energy of this rotating system?

$$K_R = \underbrace{K_1 + K_2 + K_3 + K_4}$$

translational KE

$$= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2 + \frac{1}{2} m_4 v_4^2$$

use $v = r\omega$

$$= \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \frac{1}{2} m_3 r_3^2 \omega^2 + \frac{1}{2} m_4 r_4^2 \omega^2$$

ω same for all

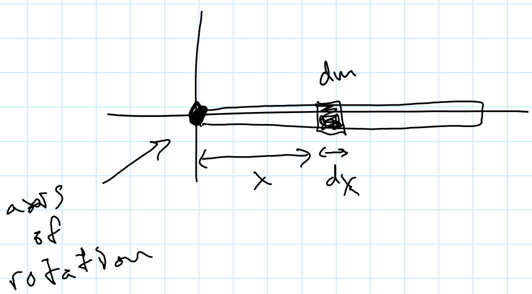
$$K_R = \frac{1}{2} \left[\sum m_i r_i^2\right] \omega^2$$

$$K = \frac{1}{2} \quad m \quad v^2$$

Inertia (pt masses)

$$K_R = \frac{1}{2} I \omega^2$$

Thin, uniform rod rotating about end pt



$$I = \int r^2 \, dm$$

$r = x$

density if uniform density

$$dm = \lambda \, dx = \frac{M}{L} dx$$

$$I = \int_0^L x^2 \frac{M}{L} dx = \frac{M}{L} \frac{x^3}{3} \Big|_0^L$$

$$= \frac{1}{3} \frac{M}{L} (L^3 - 0)$$

$$= \frac{1}{3} M L^2$$