

Goals for the Lecture:

- 1) Review momentum (1-D and 2-D collisions), problems involving both momentum and energy, center of mass, work, and energy

Power

$$P = \frac{\text{work}}{\text{time}}$$

units $\frac{\text{J}}{\text{s}} = \text{W}$
 \uparrow
 watt

Overview:

Translational Motion

Rotational Motion

Kinematics:

position
 velocity
 acceleration

$$x$$

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt}$$

$$\theta$$

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

equations

$$v_f = v_i + at$$

$$x_f = x_i + v_i t + \frac{1}{2} at^2$$

⋮

$$\omega_f = \omega_i + \alpha t$$

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$$

Newton's Laws

Inertia:

M

what changes motion:

F

2nd Law:

$$\sum \vec{F} = m\vec{a}$$

$$I = mr^2 \quad \text{for a pt. object}$$

$$\vec{\tau} \quad \text{torque}$$

$$\sum \vec{\tau} = I\vec{\alpha}$$

Energy

Kinetic Energy:

$$K = \frac{1}{2} m v^2$$

$$K_R = \frac{1}{2} I \omega^2$$

Momentum:

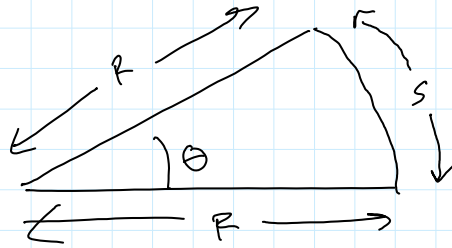
$$\vec{p} = m\vec{v}$$

$$\vec{L} = I\vec{\omega}$$

Rotation

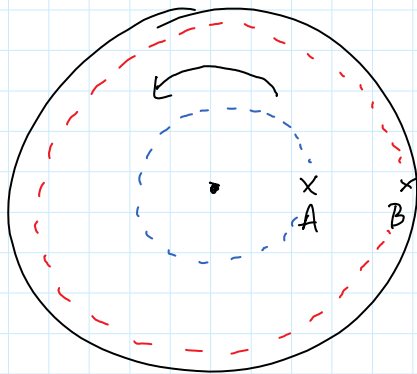
θ	ω	α
rad	$\frac{\text{rad}}{\text{s}}$	$\frac{\text{rad}}{\text{s}^2}$

rad	$\frac{\text{rad}}{\text{s}}$	$\frac{\text{rad}}{\text{s}^2}$
degrees	$\frac{\text{degrees}}{\text{s}}$	$\frac{\text{degrees}}{\text{s}^2}$
rev	$\frac{\text{rev}}{\text{s}}$	$\frac{\text{rev}}{\text{s}^2}$



$$S = R\theta$$

↑
if θ is in Radians



Who is going faster?

1) Linear or translational speed

B covers a larger circle in the same time

So,

$$V_B > V_A$$

2) Both complete a revolution in same time

So,

$$\omega_A = \omega_B$$

$$S = R\theta$$

$$V = R\omega$$

$$a = R\alpha$$

} must be

in Rad, $\frac{\text{Rad}}{\text{s}}$, $\frac{\text{Rad}}{\text{s}^2}$

Prob 10-12

... in up

slowing down

Prob 10-12

Spinning up

θ_i	0
θ_f	?
ω_i	0
ω_f	$5 \frac{\text{rev}}{\text{s}}$
α	?
t	8 s

slowing down

θ_i	? (20 rev)
θ_f	?
ω_i	$5 \frac{\text{rev}}{\text{s}}$
ω_f	0
α	?
t	12 s

← Same →

← Same →

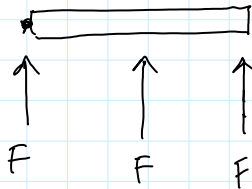
$$\begin{aligned} \theta_f &= \theta_i + \frac{1}{2}(\omega_f + \omega_i)t \\ &= 0 + \frac{1}{2}(5 + 0)8 \\ &= 20 \text{ rev} \end{aligned}$$

$$\begin{aligned} \theta_f &= \theta_i + \frac{1}{2}(\omega_f + \omega_i)t \\ &= 20 + \frac{1}{2}(5 + 0)12 \\ &= 50 \text{ rev} \end{aligned}$$

total of 50 revolutions

What causes Rotation: Torque

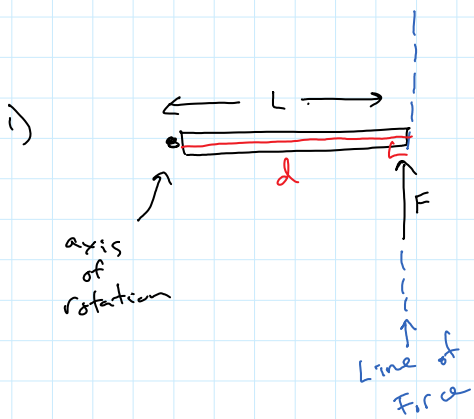
Top view of door:



Torque: $\tau = Fd$

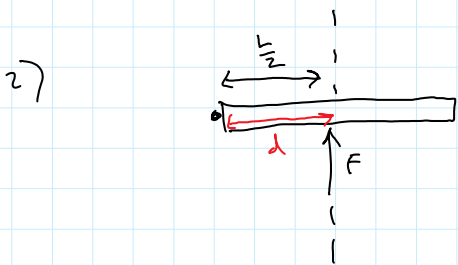
↑
 lever arm
 The perpendicular
 distance from
 line of force to
 axis of rotation

distance
of force
the axis of rotation



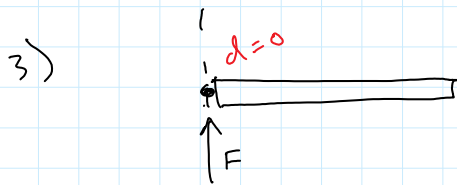
$$d = L$$

$$\tau = FL$$

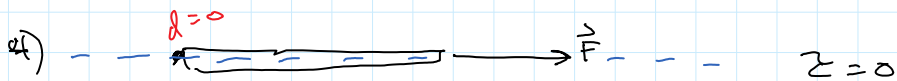


$$d = \frac{L}{2}$$

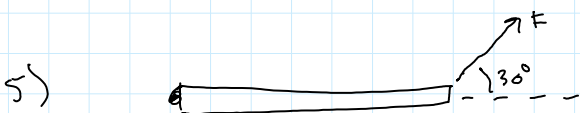
$$\tau = F \frac{L}{2}$$



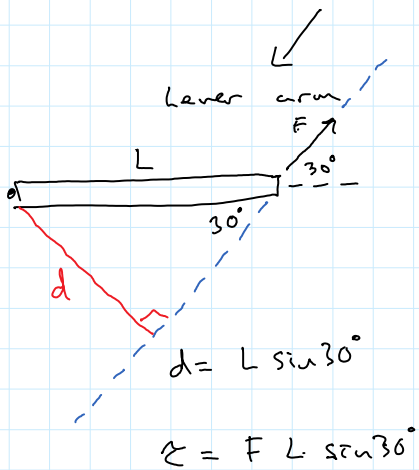
$$\tau = 0$$



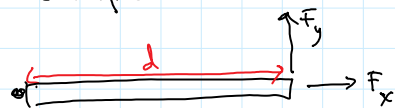
$$\tau = 0$$



2 ways to look at this



components



$$\tau = \tau_{F_x} + \tau_{F_y}$$

$$= F_y d$$

$$= F \sin \theta L$$

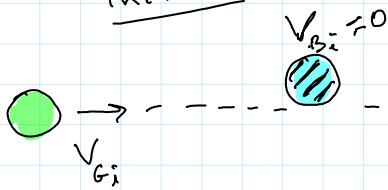
$$= FL \sin 30^\circ$$

Torque

$$\tau = Fd$$

units: Nm [Not J (joules)]

2-D Collision
initial



$$M_G = M_B = m$$

given everything except θ_B and θ_G , find θ_B and θ_G

$$\vec{p}_i = \vec{p}_f$$

$$(p_x)_i = (p_x)_f \rightarrow +$$

$$m v_{Gi} + 0 = m v_{Gf} \cos \theta_G + m v_{Bf} \cos \theta_B$$

$$v_{Gi} = v_{Gf} \cos \theta_G + v_{Bf} \cos \theta_B$$

$$(p_y)_i = (p_y)_f \uparrow +$$

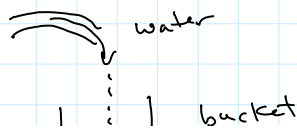
$$0 = m v_{Bf} \sin \theta_B - m v_{Gf} \sin \theta_G$$

$$v_{Bf} \sin \theta_B = v_{Gf} \sin \theta_G$$

solve for θ_B and θ_G

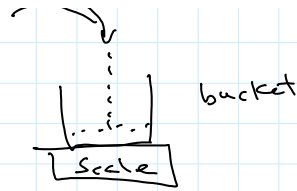
Bank Prob

9-21



Book Prob

9-21



$$N_{\text{scale}} = (\text{Weight of bucket}) + (\text{Weight of water}) + \frac{\text{impulse}}{\text{time}}$$

$$I = F \Delta t = \Delta p$$

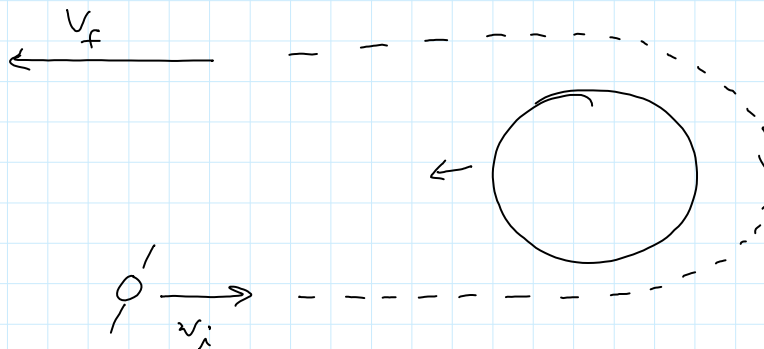
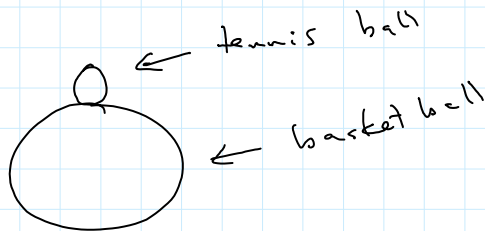
$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta (mv)}{\Delta t} = \left(\frac{m}{\Delta t} \right) \Delta v$$

$$= \left(0.25 \frac{\text{L}}{\text{s}} \right) \left(\rho \frac{\text{kg}}{\text{L}} \right) \Delta v$$

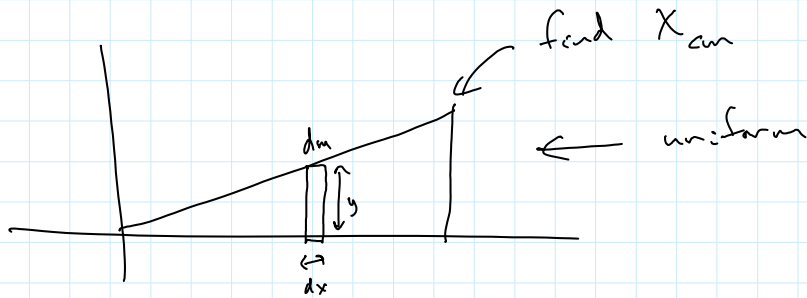
↑
density
of
water

↑
use
kinematics

$$\rho = 1 \frac{\text{kg}}{\text{L}}$$

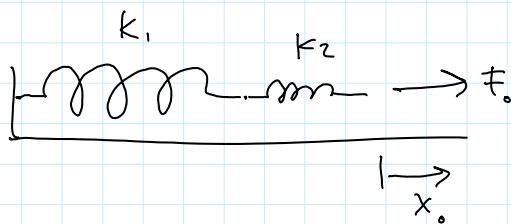


$$|\vec{v}_f| > |\vec{v}_i|$$



$$dm = \sigma dA = \sigma y dx$$

$$= \sigma (mx + b) dx$$



Spring 1 stretches x_1

Spring 2 stretches x_2

$x_1 + x_2 = x_0$

$$(F_{sp})_1 = (F_{sp})_2 = F_0$$

$$k_1 x_1 = k_2 x_2 = k_{eq} x_0$$

$$F = () x_0$$

↑
eq. Spring constant

$$x_1 + x_2 = x_0$$

$$\frac{k_2 x_2}{k_1} + x_2 = x_0$$

$$\left(\frac{k_2}{k_1} + 1 \right) x_2 = x_0$$

$$\left(\frac{k_2}{k_1} + 1\right) X_2 = X_0$$

$$X_2 = \frac{1}{\left(\frac{k_2}{k_1} + 1\right)} X_0$$

$$k_2 X_2 = \frac{k_2}{\left(\frac{k_2}{k_1} + 1\right)} X_0$$

$$F = \frac{k_2}{\frac{k_2}{k_1} + 1} X_0$$

$$F = \frac{k_1 k_2}{\underbrace{k_1 + k_2}_{k_{eq}}} X_0$$