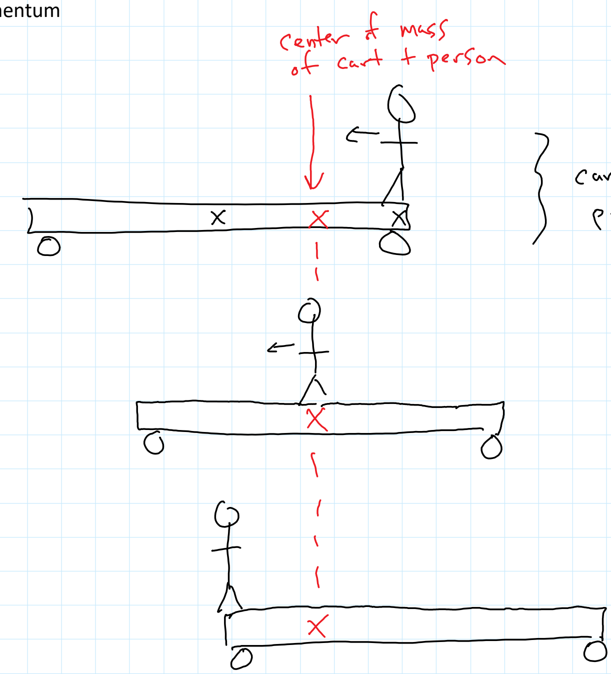
Phy 4A 11/7

Monday, November 7, 2016 9:55 AM

Goals for the Lecture:

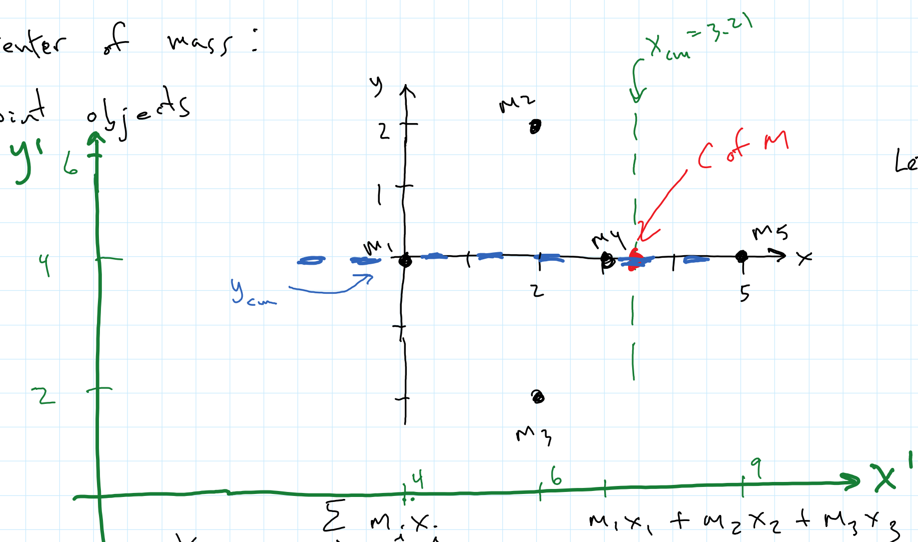
1) Calculate the center of mass for point objects and solid objects
2) Use center of mass to solve problems
3) Solve 2-D collision problems using momentum



cart is at rest
person begins walking to other side

Find center of mass:

Point objects



Let
$m_1 = 1\text{ kg}$
$m_2 = m_3 = 2\text{ kg}$
$m_4 = 4\text{ kg}$
$m_5 = 5\text{ kg}$

$$X_{cm} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4 + m_5x_5}{m_1 + m_2 + m_3 + m_4 + m_5}$$

$$= \frac{(1\text{ kg})(0\text{ m}) + (2\text{ kg})(2\text{ m}) + (2\text{ kg})(2\text{ m}) + (4\text{ kg})(3\text{ m}) + (5\text{ kg})(5\text{ m})}{(1+2+2+4+5)\text{ kg}}$$

$$= \frac{0 + 4 + 4 + 12 + 25}{14}\text{ m}$$

$$= 3.21\text{ m}$$

$$y_{cm} = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{(1\,kg)(0\,m) + (2\,kg)(2\,m) + (2\,kg)(-2\,m) + (4\,kg)(0) + (5\,kg)(0)}{14}$$

$$= 0$$

Solid objects:

$$x_{cm} = \frac{\int x\,dm}{\int dm}$$

1-D object (rod) uniform density

$$dm = \frac{M}{L}\,dx$$

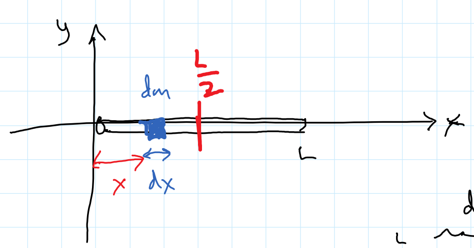
↑ mass density

$$dm = \lambda\,dx \qquad \text{in general}$$

↑ mass density

	Total mass		uniform density	density	units
1-D	M	length L	$\frac{M}{L}$	λ	$\frac{kg}{m}$
2-D	M	Area A	$\frac{M}{A}$	σ	kg/m^2
3-D	M	Volume V	$\frac{M}{V}$	ρ	$\frac{kg}{m^3}$

Find c of m of uniform density Rod, length L
mass M

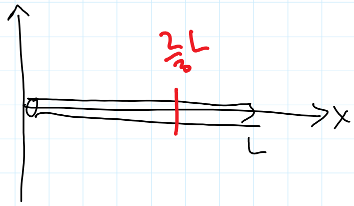


$$x_{cm} = \frac{\int x\,dm}{\int dm}$$

$$x_{cm} = \frac{\int_0^L x\,\overbrace{(\lambda\,dx)}^{dm}}{\int_0^L \lambda\,dx} = \frac{\lambda \int_0^L x\,dx}{\lambda \int_0^L dx} = \frac{\lambda \left.\frac{x^2}{2}\right|_0^L}{\lambda \left. x\right|_0^L}$$

$$= \frac{\frac{\lambda}{2} L^2}{\lambda L} = \frac{\frac{M}{2L} L^2}{\frac{M}{L} L} = \frac{\frac{1}{2} ML}{M} = \frac{L}{2}$$

Non-uniform Rod:

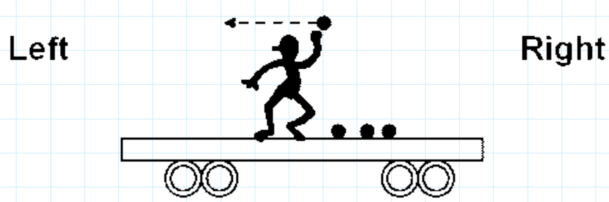


$\lambda = \alpha x$ α is a positive constant

$$X_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\int_0^L x\,(\lambda\,dx)}{\int_0^L \lambda\,dx} = \frac{\int x\,(\alpha x\,dx)}{\int \alpha x\,dx} = \frac{\cancel{\alpha}\int_0^L x^2\,dx}{\cancel{\alpha}\int_0^L x\,dx}$$

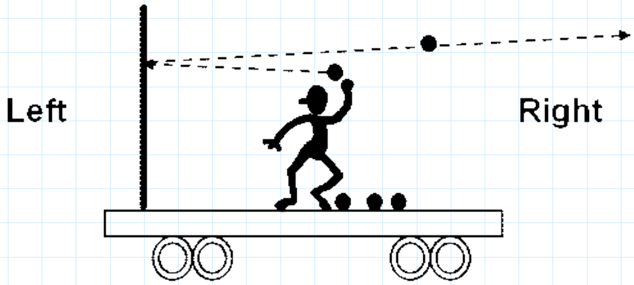
$$= \frac{\left.\frac{x^3}{3}\right|_0^L}{\left.\frac{x^2}{2}\right|_0^L} = \frac{2}{3}L$$

Suppose you are on a cart initially at rest that rides on a frictionless track. If you throw a ball off the cart towards the left, will the cart be put into motion?



1. $\boxed{1.}$ Yes, and it moves to the right.
2. Yes, and it moves to the left.
3. No, it remains in place.

Suppose you are on a cart which is initially at rest that rides on a frictionless track. You throw a ball at a vertical surface that is firmly attached to the cart. If the ball bounces straight back as shown in the picture, will the cart be put into motion after the ball bounces back from the surface?



1. Yes, and it moves to the right
2. Yes, and it moves to the left.
3. No, it remains in place.

Now, with some changes:

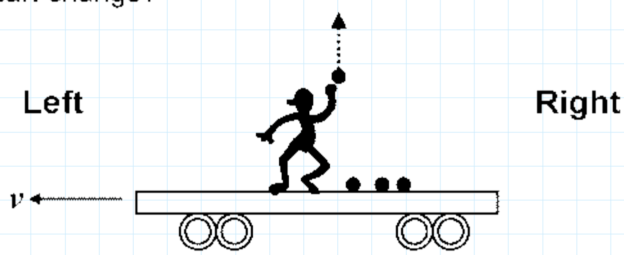
Time t_0 : ball leaves hand

t_1 : ball comes to rest against wall

t_2 : person catches ball

time interval	direction of cart
$t_0 \rightarrow t_1$	to the right
at t_1	stopped
$t_1 \rightarrow t_2$	to the left
at t_2	stopped

Suppose you are on a cart that is moving at a constant speed v toward the left on a frictionless track. If you throw a massive ball straight up (from your perspective), how will the speed of the cart change?



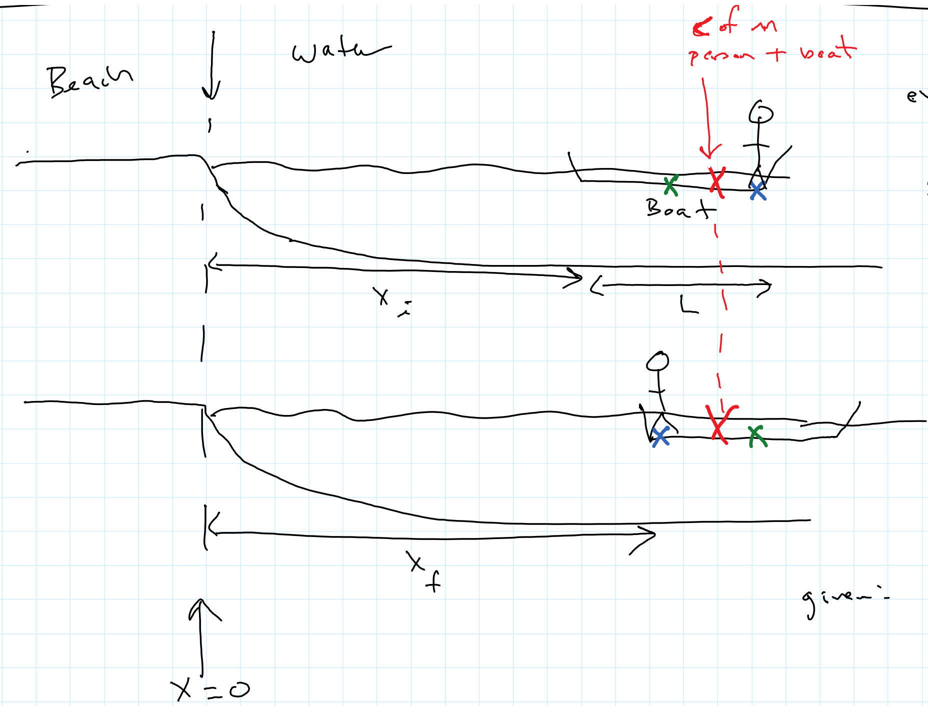
1. Increase
2. Decrease
3. Will not change
4. You need to know how fast you throw the ball

Example Beach water

C of M
person + boat

everything at rest

Example



everything at rest

person walks across boat

boat moves away from beach

everything at rest again

given: $x_i = 10$ m

$m_{person} = 100$ kg

$M_{boat} = 200$ kg

$L = 2$ m

boat is uniform mass density

Find: how much boat moves

C of M of boat + person will Not change

$$(X_{cm})_i = (X_{cm})_f$$

$$\left(\frac{\sum m_i x_i}{\sum m_i}\right)_i = \left(\frac{\sum m_i x_i}{\sum m_i}\right)_f$$

$$\left(\frac{m_p x_p + m_b x_b}{m_p + m_b}\right)_i = \left(\frac{m_p x_p + m_b x_b}{m_p + m_b}\right)_f$$

$$\frac{(100\text{ kg})(10+2) + (200)(10+1)}{\cancel{100+200}} = \frac{(100)\, x_f + 200\,(x_f + 1)}{\cancel{300}}$$

$$x_f + \frac{2}{3} = 11\frac{1}{3}$$

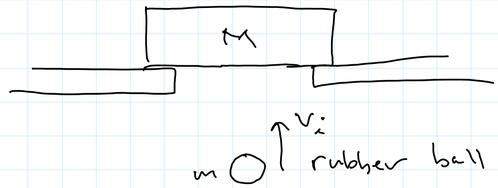
$$x_f = 10\frac{2}{3}\text{ m}$$

$\Delta x = \quad 2 \quad$ m

$x_f = \frac{10}{3}$ m

$$\Delta X = \frac{2}{3} \text{ m}$$

Like Problem 9-71 (with changes)



given: $m = 0.3$ kg

$M = 4$ kg

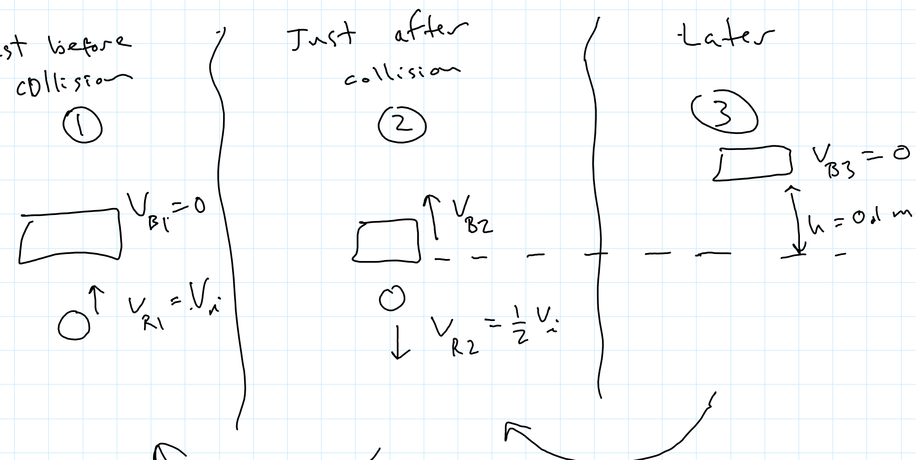
M goes up 0.1 m after collision

m rebounds with $\frac{1}{2}$ its initial speed

find: a) v_i of the rubber ball, just before the collision

b) is it an elastic collision?

Time line:



$2 \to 1$
use momentum to find v_i

$3 \to 2$
use energy to find V_{B2}

$$E_3 = E_2$$

$$\cancel{M}gh = \frac{1}{2}\cancel{M}V_{B2}^2$$

$$V_{B2} = \sqrt{2gh}$$

if $g = 10 \frac{m}{s^2}$

$$= \sqrt{2}$$

$$= 1.4$$

$$p_1 = p_2 \quad \uparrow$$

$$m V_{R1} + 0 = m\left(\tfrac{1}{2} V_{R1}\right) + M V_{B2}$$

$$m\left(\tfrac{3}{2} V_{R1}\right) = M\sqrt{2}$$

$$V_{R1} = \frac{M}{m}\,\frac{2\sqrt{2}}{3}$$

$$= \frac{4}{0.3}\,\frac{2\sqrt{2}}{3}$$

$$= 12.4 \ \frac{m}{s}$$

b)

$$K_i \overset{?}{=} K_f$$

$$\tfrac{1}{2} m V_{R1}^2 + 0 \overset{?}{=} \tfrac{1}{2} m V_{R2}^2 + \tfrac{1}{2} M V_{B2}^2$$

$$\tfrac{1}{2} m V_{R1}^2 \overset{?}{=} \tfrac{1}{2} m \left(\frac{V_{R1}}{2}\right)^2 + \tfrac{1}{2} M V_{B2}^2$$

$$\tfrac{1}{2}(0.3)(12.4)^2 \overset{?}{=} \tfrac{1}{2}(0.3)(6.2)^2 + \tfrac{1}{2}(4)(\sqrt{2})^2$$

$$23 \overset{?}{=} 6 + 4$$

$$23 > 10$$

It is <u>Inelastic</u>

since $K_i \neq K_f$