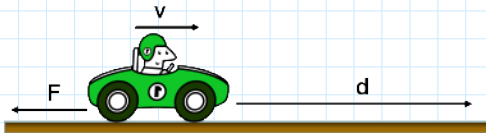


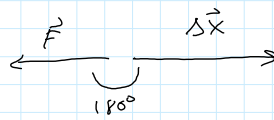
**Goals for the Lecture:**

- 1) Be able to calculate work done by constant and variable forces
- 2) Be able to calculate the scalar product (dot product) of two vectors
- 3) Understand how defining your system can affect the work done on the system or by the system

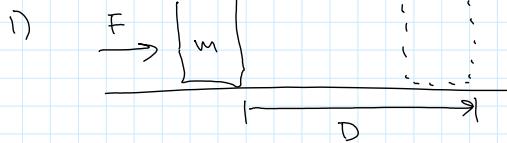
A car traveling to the right with a speed  $v$  brakes to a stop in a distance  $d$ . What is the work done on the car by the frictional force  $F$ ? (Assume that the frictional force is constant).



1.  $W = F \cdot d$
2.  $W = -F \cdot d$
3.  $W = 0$
4.  $W = F \cdot v$
5.  $W = -F \cdot v$

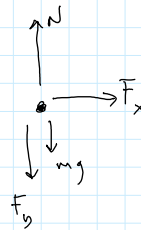
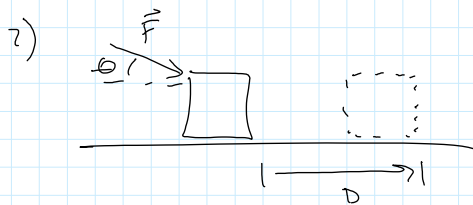


Examples



$W = FD$

since  $F$  is constant and it points in same direction as  $\Delta \vec{x}$



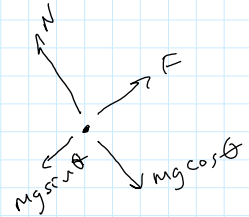
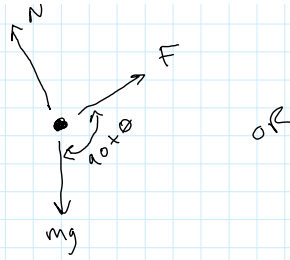
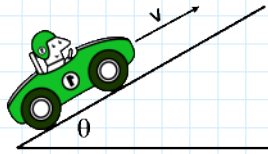
$W = F_x D = F \cos \theta D$

In general: (constant forces)  $W = \vec{F} \cdot \Delta \vec{x}$   
 $W = |\vec{F}| |\Delta \vec{x}| \cos \theta$

units for work: joule (J)

A car travels with a constant velocity  $v$  for a distance  $d$  up a hill that makes an angle  $\theta$  with respect to the horizontal. What is the work done by the sum of the gravitational force plus the constant upward frictional force  $F$  that the hill exerts on the tires of the car?





1.  $W = (mgsin\theta)d$
2.  $W = -(mgsin\theta)d$
3.  $W = 0$
4.  $W = Fd$
5.  $W = -Fd$

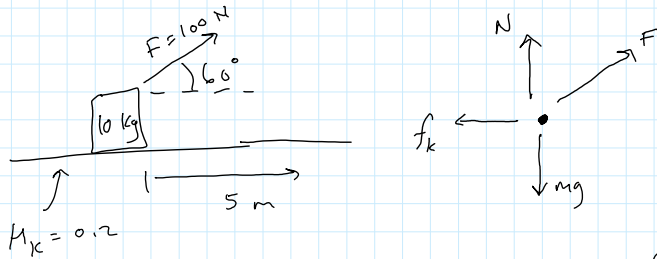
Force	Work done on car
N	$W = N(\Delta x) \cos 90^\circ = 0$
F	$F \Delta x \cos 0^\circ = Fd$
gravity	$mg(\Delta x) \cos(90+\theta) = -mg \sin\theta d$
$F_{net}$	$W_N + W_F + W_{mg} = 0 + Fd - mg \sin\theta d$

must be zero because speed is not changing

Kinetic energy:  $K = \frac{1}{2} m v^2$

Work-KE Theorem:  $\sum W = \Delta K$

Example



Find the work done by every force acting on the box and the Net work done on the box

Force	Work done on the box
N	$W_N = N(\Delta x) \cos 90^\circ = 0$
Mg	$W_{mg} = (mg)(\Delta x) \cos 90^\circ = 0$
F	$W_F = F(\Delta x) \cos 60^\circ = 250 \text{ J}$
$f_k$	$W_{f_k} = f_k(\Delta x) \cos 180^\circ = \mu_k N \Delta x (-1) = -14 \text{ J}$
Net Work	$W_{Net} = W_N + W_{mg} + W_F + W_{f_k}$

Find  $N$ :

$$\sum F_y = 0$$

$$N + F \sin\theta - mg = 0$$

$$N = mg - F \sin\theta$$

$$= 100 - 86 = 14$$

$$= 0 + 0 + 250 - 14$$

$$= +236 \text{ J}$$

if box started from rest, how fast is it moving after 5 m?

$$W_{\text{Net}} = \Delta K$$

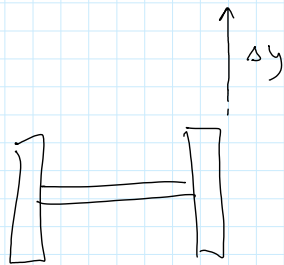
$$= K_f - K_i$$

$$= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$236 = \frac{1}{2} (10) v_f^2$$

$$v_f = 6.9 \text{ m/s}$$

Lifting weights:



Work done by you on the barbell  
(Lift at constant speed)

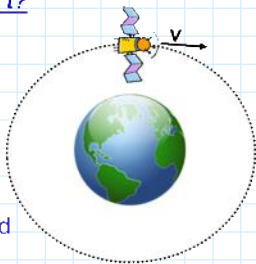
$$W_{\text{up}} = mg \Delta y \cos 0^\circ = mg \Delta y$$

Lower the barbell

$$W_{\text{down}} = mg \Delta y \cos 180^\circ = -mg \Delta y$$

$$W_{\text{net}} = 0$$

A satellite travels with a constant speed  $|v|$  as it moves around a circle centered on the earth. How much work is done by the gravitational force  $F$  on the satellite **after it travels half way around the earth in time  $t$** ?



1. Cannot be determined

2.  $W = 0$

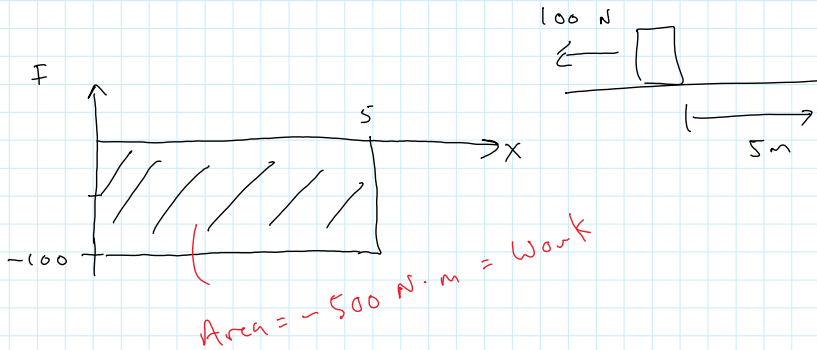
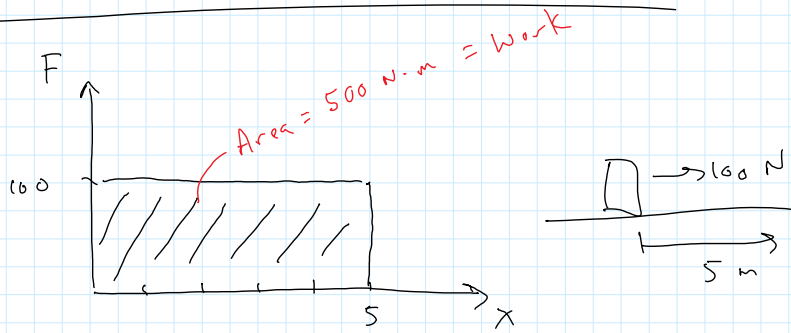
3.  $W = F \cdot |v|t$

4.  $W = -F \cdot |v|t$

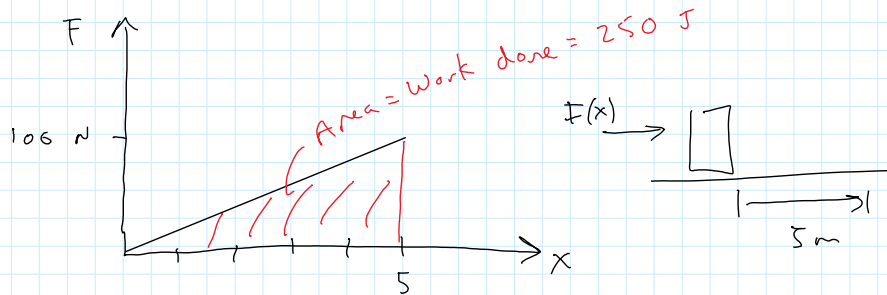
Because  $F_g \perp \Delta x$

$\cdot = \text{work}$

4.  $W = -\int \mathbf{F} \cdot d\mathbf{r}$

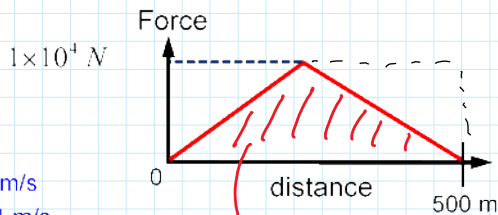


if I give you F vs X graph, find work done:



find work done on box:  $W = \frac{1}{2} (100)(5) = 250 \text{ J}$

The engine of a 1000 kg sports car rotates the tires, creating a forward pushing force  $F$  on the tires of the car that varies as a function of distance. The force is shown below. If the car starts at rest, what is the speed of the car after traveling 500 meters?



1. 0 m/s
2. 71 m/s
3. 100 m/s
4. 141 m/s
5. 200 m/s

Area =  $\frac{1}{2} (10,000)(500) = 2,500,000 = W_{\text{net}}$

$W_{\text{net}} = \Delta K$   
 $= \frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2$

5. 200 m/s

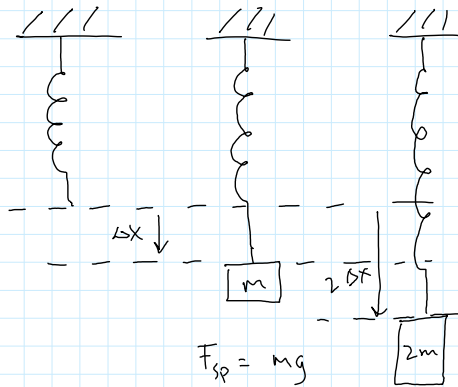
$$W_{\text{net}} = \Delta K$$

$$= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$2,500,000 = \frac{1}{2} (1000) v_f^2$$

$$v_f = 71 \frac{\text{m}}{\text{s}}$$

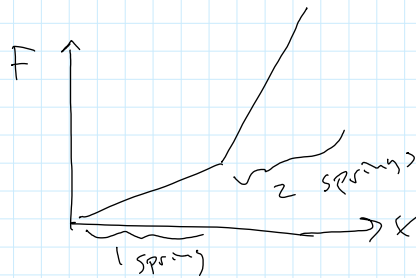
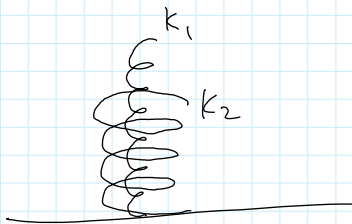
Springs:



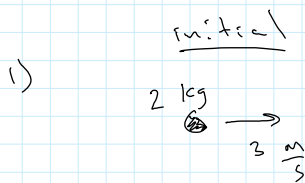
Hooke's Law

$$F_{\text{sp}} = k(\Delta X)$$

↑  
Spring constant



Find the change in KE of each ball:



$$\Delta K = K_f - K_i$$

$$= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$= \frac{1}{2} (2)(1)^2 - \frac{1}{2} (2)(3)^2$$

$$= 1 - 9$$

$$= -8 \text{ J}$$

2) 2 kg



$$\Delta K = K_f - K_i$$

$$= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$= 25 - 9$$

$$= 16 \text{ J}$$

3) 2 kg



$$\Delta K = K_f - K_i$$

$$= 9 - 9$$

$$= 0 \text{ J}$$

These 2 together are the same as 3)

4)

2 kg



$$\Delta K = K_f - K_i$$

$$= 0 - 9$$

$$= -9 \text{ J}$$

5)



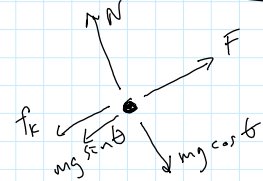
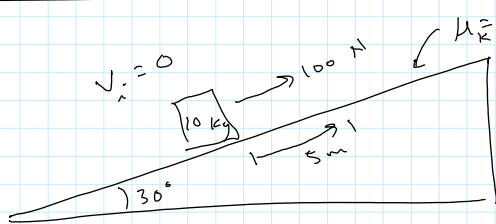
$$\Delta K = K_f - K_i$$

$$= 9 - 0$$

$$= +9 \text{ J}$$

together 0 J

Example



Find the work done by every force acting on the box and the final speed of the box

Force	Work Done on Box
$N$	0
$f_k$	$f_k (\Delta x) \cos 180^\circ$ $\mu_k (mg \cos 30^\circ) (5) (-1)$ $-87 \text{ J}$

	$\mu_k (mg \cos 30^\circ) (5) (-1)$ $-87 \text{ J}$
$mg \sin \theta$	$(mg \sin \theta) (\Delta x) \cos 180^\circ$ $50 (5) (-1)$ $-250 \text{ J}$
$mg \cos \theta$	$0$
$F$	$(100) (5) (\cos 0^\circ)$ $500 \text{ J}$

$$W_{\text{net}} = 0 - 87 + 0 - 250 + 500$$

$$= +163 \text{ J}$$

$$W_{\text{net}} = \Delta K$$

$$= K_f - K_i$$

$$= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$163 = \frac{1}{2} (10) v_f^2$$

$$v_f = 5.7 \frac{\text{m}}{\text{s}}$$