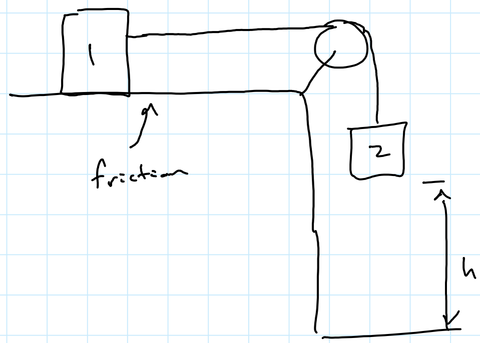
Phy 2A Review 6/9

Friday, June 9, 2017 10:03 AM

Rotational Energy Problems:

1)



given: $m_1 = 5$ kg

$m_2 = 2$ kg

$m_{pulley} = 0.5$ kg

$R_{pulley} = 0.3$ m

Pulley is a solid disk

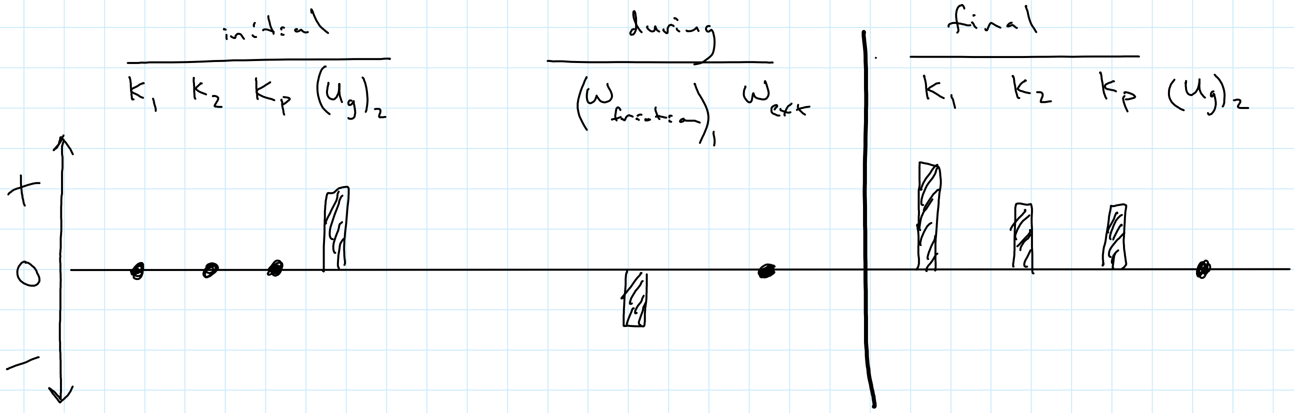
$\mu_k = 0.2$

$V_i = 0$

$h = 1.5$ m

find V_f (speed of m_2 just before it hits the ground)

use energy conservation:



$$m_2 g h - \mu_k \underbrace{(m_1 g)}_{N} h = \frac{1}{2} m_1 V_f^2 + \frac{1}{2} m_2 V_f^2 + \underbrace{\frac{1}{2} I \omega_f^2}_{K_p}$$

solid disk: $I = \frac{1}{2} M_p R_p^2$

since $V = r\omega$

$$\omega = \frac{V}{R_p}$$

$$m_2 g h - \mu_k m_1 g h = \frac{1}{2} m_1 V_f^2 + \frac{1}{2} m_2 V_f^2 + \frac{1}{2}\left(\frac{1}{2} M_p R_p^2\right)\left(\frac{V_f}{R_p}\right)^2$$

$$m_2 g h - \mu_k m_1 g h = \frac{1}{2} m_1 v_f^2 + \frac{1}{2} m_2 v_f^2 + \frac{1}{4} M_p v_f^2$$

$$m_2 g h - \mu_k m_1 g h = \frac{1}{2}\left(m_1 + m_2 + \frac{M_p}{2}\right) v_f^2$$

$$(2)(9.8)(1.5) - (0.2)(5)(9.8)(1.5) = \frac{1}{2}\left(5 + 2 + \frac{0.5}{2}\right) v_f^2$$

$$14.7 = 3.625\, v_f^2$$

$$v_f = 2.01\ \frac{m}{s}$$

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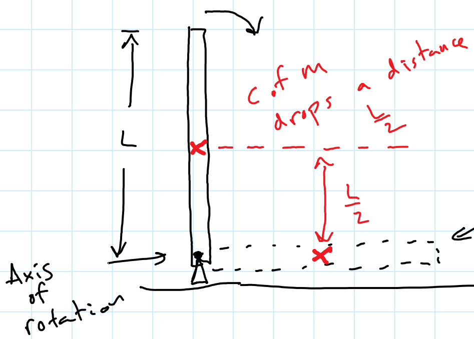
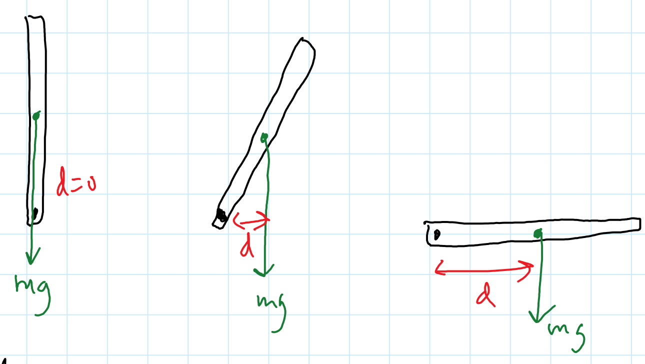
2) A uniform rod can rotate about an axis of rotation through one end of the rod. It start vertical and rotates / falls over. It is horizontal the instant before it hits the ground.



given: $m = 2$ kg
$L = 1.5$ m
$\omega_i = 0$

find ω_f
(rotational speed just before it hits the ground)

Because the torque changes as the rod falls, the angular acceleration, α, is Not constant. So, we cannot use rotational kinematics.



$$\tau = mg\, d$$ lever arm

Use energy:

$$E_i = E_f$$ (No friction or external forces)

$$\cancel{K_R}^{\,0} + U_g = K_R + \cancel{U_g}^{\,0}$$

$$mgh = \frac{1}{2} I \omega_f^2$$

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distance the c of m drops $\left(h = \frac{L}{2}\right)$

$I = \frac{1}{3} mL^2$ rod rotating about its end point

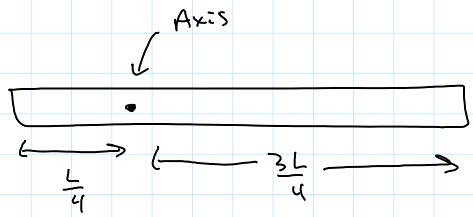
$$mg\frac{L}{2} = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega_f^{\,2}$$

$$2(9.8)\left(\frac{1.5}{2}\right) = \frac{1}{2}\left[\frac{1}{3}(2)(1.5)^2\right]\omega_f^{\,2}$$

$$14.7 = 0.75\ \omega_f^{\,2}$$

$$\omega_f = 4.43\ \frac{rad}{s}$$

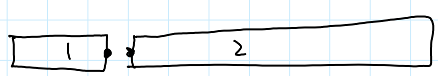
Find I for a rod rotating about this axis:



total length = L

total mass = M

Like 2 Rods (each rotating about their end points)



Rod 1: $m_1 = \frac{M}{4}$

$L_1 = \frac{L}{4}$

Rod 2: $m_2 = \frac{3M}{4}$

$L_2 = \frac{3L}{4}$

$$I_1 = \frac{1}{3}m_1L_1^2 = \frac{1}{3}\left(\frac{M}{4}\right)\left(\frac{L}{4}\right)^2 = \frac{1}{192}ML^2$$

$$I_2 = \frac{1}{3}m_2L_2^2 = \frac{1}{3}\left(\frac{3M}{4}\right)\left(\frac{3L}{4}\right)^2 = \frac{27}{192}ML^2$$

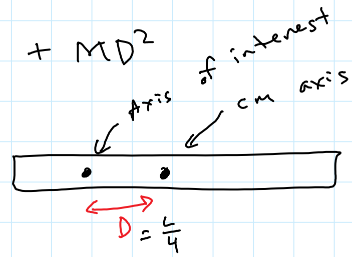
$$I_{total} = I_1 + I_2 = \frac{28}{192}ML^2 = \frac{7}{48}ML^2$$

OR, use the Parallel axis theorem:

$I = I + MD^2$ ... terest

OR, use the Parallel axis theorem:

$$I = I_{cm} + MD^2$$

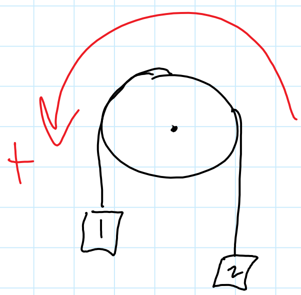


$$= \frac{1}{12} ML^2 + M\left(\frac{L}{4}\right)^2$$

$$= \frac{7}{48} ML^2$$

Newton's Law Problem:

Find the angular acceleration of this pulley:



$M_1 = 5$ kg

$M_2 = 3$ kg

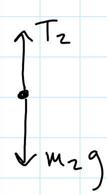
$M_p = 2$ kg

$R_p = 0.4$ m

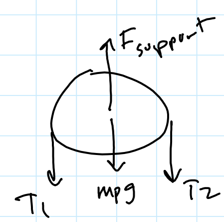
Pulley is a solid disk

m_1

M_2



Pulley



$\Sigma \vec{F}_1 = m_1 \vec{a}$ $\downarrow +$

$m_1 g - T_1 = m_1 a$

$T_1 = m_1 g - m_1 a$

$\Sigma \vec{F}_2 = m_2 \vec{a}$ $\uparrow +$

$T_2 - m_2 g = m_2 a$

$T_2 = m_2 a + m_2 g$

$\Sigma \vec{\tau} = I \vec{\alpha}$ $+$

$T_1 R_p - T_2 R_p = \left(\frac{1}{2} m_p R_p^2\right) \alpha$

$a = r\alpha$

$\alpha = \frac{a}{R_p}$

$T_1 - T_2 = M_p a$

$$T_1 - T_2 = \frac{m_p}{2} a$$

$$(m_1 g - m_1 a) - (m_2 a + m_2 g) = \frac{m_p}{2} a$$

$$m_1 g - m_2 g = \left(m_1 + m_2 + \frac{m_p}{2}\right) a$$

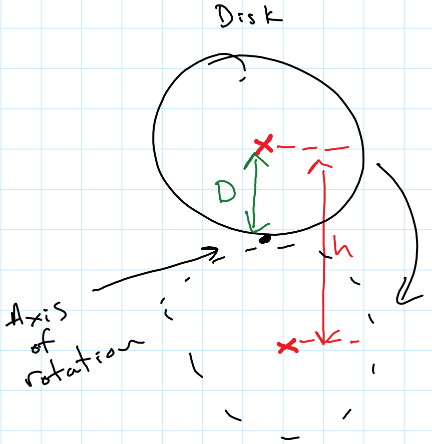
$$(5 - 3)\, 9.8 = \left(5 + 3 + \frac{2}{2}\right) a$$

$$\frac{19.6}{9} = a$$

$$a = 2.18 \ \frac{m}{s^2}$$

$$\alpha = \frac{a}{R_p} = \frac{2.18}{0.4} = 5.44 \ \frac{rad}{s^2}$$

Energy Problem



given: $M = 2$ kg
$R = 0.5$ m
solid disk
release from rest

find ω_f (when it is at its lowest point)

$$E_i = E_f \quad \text{(No friction)}$$

$$mgh = \frac{1}{2} I \omega_f^2$$

C of M drops by 2R

Need Parallel axis theorem:

$$I = I_{cm} + MD^2$$
$$= \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

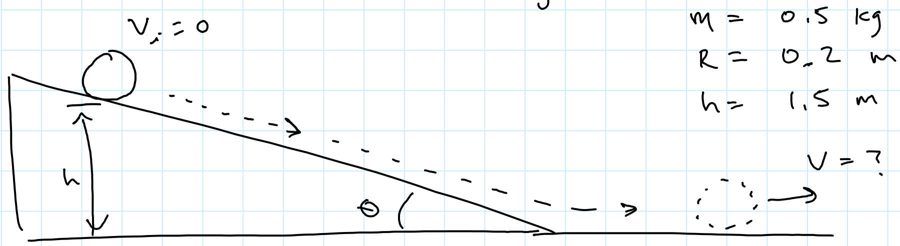
$$mg(2R) = \frac{1}{2}\left(\frac{3}{2}MR^2\right)\omega_f^2$$

$$\frac{8}{3}\frac{g}{R} = \omega_f^2$$

$$\omega_f = \sqrt{\frac{8(9.8)}{3(0.5)}} = 7.23 \ \frac{rad}{s}$$

Ball rolling down a hill

given: $v_i = 0$
hollow ball
$m = 0.5$ kg
$R = 0.2$ m
$h = 1.5$ m



find v_f

$$E_i = E_f$$

$$mgh = K_{translational} + K_{rotational}$$

$$= \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2$$

$$v_f = R\,\omega_f$$

$$\cancel{m} gh = \frac{1}{2} \cancel{m} v_f^2 + \frac{1}{2}\left(\frac{2}{3} \cancel{m} R^2\right)\left(\frac{v_f}{R}\right)^2$$

$$gh = \left(\frac{1}{2} + \frac{1}{3}\right) v_f^2$$

$$v_f = 4.2 \ \frac{m}{s}$$