

Goals for the Lecture:

- 1) Understand Newton's Universal Law of Gravity and how to use it to solve problems
- 2) Understand gravitational potential energy when objects are not close to the earth's surface
- 3) Understand escape speed and be able to calculate it

Potential Energy:

gravity on Earth:

$$F_g = mg$$

$$W_g = F_g y = mgy$$

$$U_g = mgy$$

Spring:

$$F_{sp} = kx$$

$$W = \int F dx$$

$$W_{sp} = \frac{1}{2} kx^2$$

$$U_{sp} = \frac{1}{2} kx^2$$

Universal gravity:

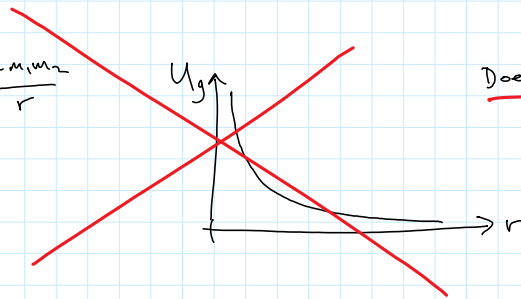
$$F = \frac{G m_1 m_2}{r^2}$$

$$\text{using } W = \int F dr$$

$$W = - \frac{G m_1 m_2}{r}$$

$$U_g = - \frac{G m_1 m_2}{r}$$

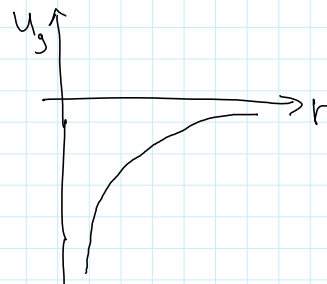
$$\text{if } U_g = + \frac{G m_1 m_2}{r}$$

Does Not work

U must go down
as r decreases

U must increase
as r increases

$$\text{if } U_g = - \frac{G m_1 m_2}{r}$$

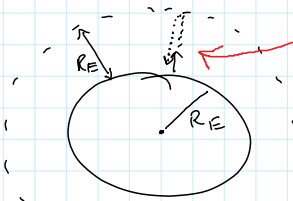


this works!

- Negative is okay
 - zero is maximum PE
 - cannot define zero anywhere
- $U_g = 0$ only at $r = \infty$

$U_g = 0$ only at $r = \infty$
 you cannot define zero to any other location

Problem: Throw a ball straight up so it reaches a height of R_E above the surface of the earth (No air resistance)



this is a large distance, so Force of gravity decreases (it is not constant over the entire path)

cannot use kinematics, because F_g changes over this large distance (a is not constant)
 \rightarrow must use energy

so, do not use kinematics \rightarrow use energy

$$E_i = E_f$$

$$K_i + (U_g)_i = K_f + (U_g)_f$$

$$\frac{1}{2} m v_i^2 - \frac{G M M_E}{R_E} = \frac{1}{2} m v_f^2 - \frac{G M M_E}{2 R_E}$$

mass of ball
 mass of earth

$$\frac{1}{2} v_i^2 = \frac{G M M_E}{2 R_E}$$

$$v_i = \sqrt{\frac{G M_E}{R_E}}$$

Prob: Find the escape speed from Earth's surface:

$$E_i = E_f$$

$$K_i + (U_g)_i = K_f + (U_g)_f$$

$$\frac{1}{2} m v_i^2 - \frac{G m_1 M_E}{R_E} = 0 + 0$$

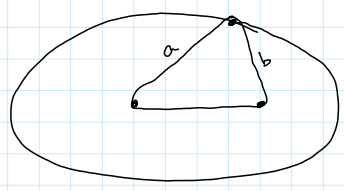
$\swarrow r = \infty, \text{ so } U_g = 0$
 \uparrow object comes just to rest at $r = \infty$

$$\frac{1}{2} m v_i^2 = \frac{G m_1 M_E}{R_E}$$

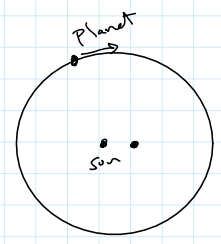
$$v_i = \sqrt{2 \frac{G M_E}{R_E}}$$

Kepler's Laws (Not on the exam)

1) Planets move in elliptical orbits with the sun at one focus

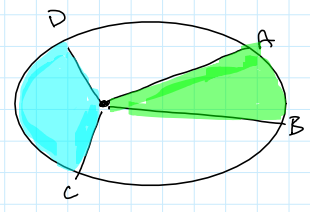


$a + b = \text{constant}$



just a slight ellipse

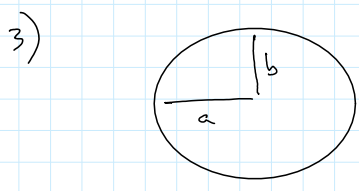
2)



$$(\text{Area})_{AB} = (\text{Area})_{CD}$$

So, planets speed up when close to the sun and slow down when far from the sun

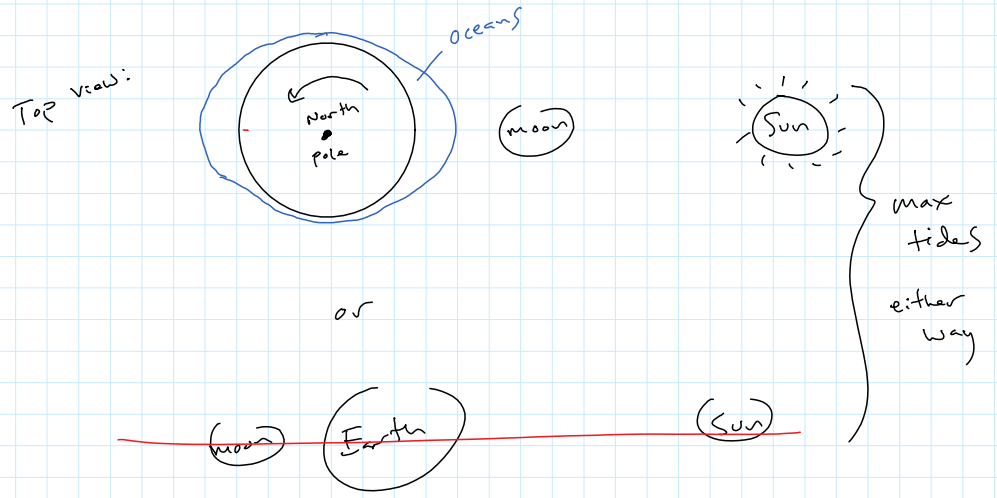
from the sun



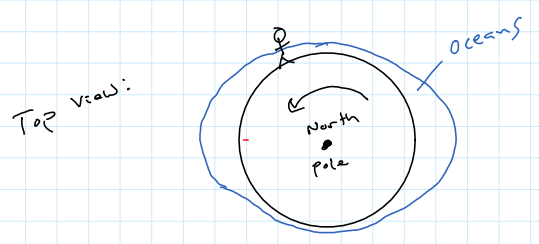
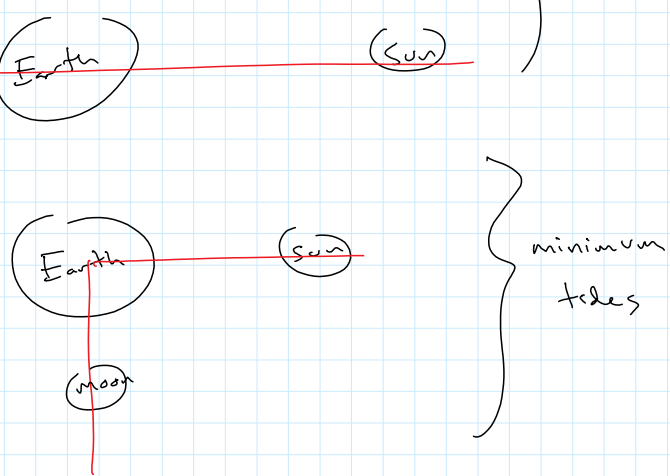
$T =$ period (time it takes to complete one revolution)

$$T^2 \propto a^3$$

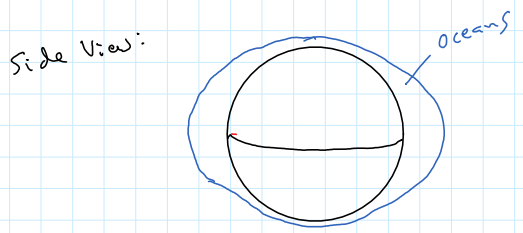
Tides:



or



as the earth rotates the water levels rise and fall

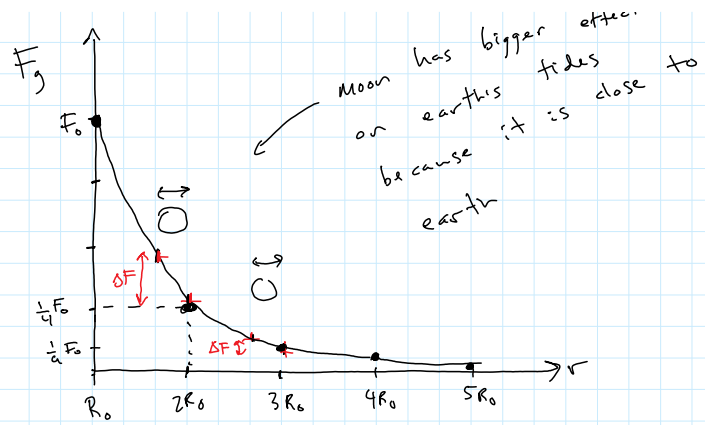


$$F = \frac{GmM_E}{r^2}$$

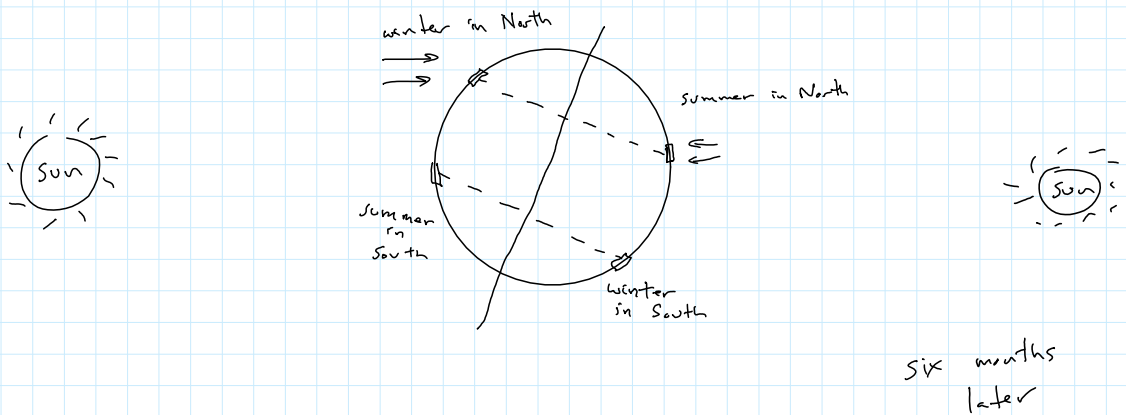
$F_g \uparrow$

- moon has bigger effect this tides close to

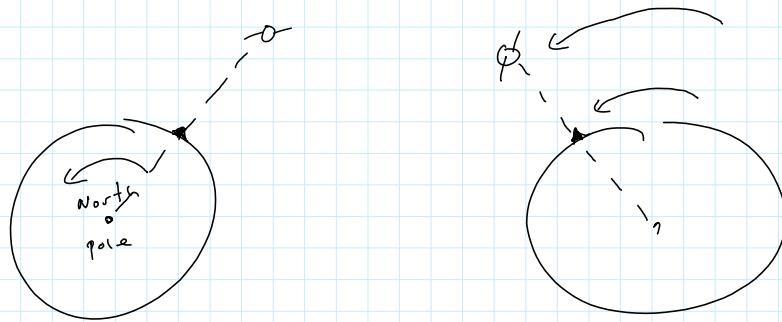
$$F = \frac{GmM_E}{r^2}$$



Seasons:



Geosynchronous orbit



Find r (distance from center of earth)
for geosynchronous orbit

circular motion:

$$\Sigma F_{\text{radial}} = m a_{\text{app}}$$

$$\frac{GM_E}{r^2} = m \frac{v^2}{r}$$

mass of satellite
cancels out

what is v ?

$$v = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{(1 \text{ day}) \left(\frac{24 \text{ hr}}{\text{day}} \right) \left(\frac{3600 \text{ s}}{\text{hr}} \right)}$$
$$= \frac{2\pi r}{(24)(3600)}$$

$$\frac{GM_E}{r^2} = \frac{1}{r} \left[\frac{2\pi}{(24)(3600)} r \right]^2$$

$$\frac{GM_E}{\left[\frac{2\pi}{24(3600)} \right]^2} = r^3$$

