Phy 2A - Gravity Problems

Sunday, March 26, 2017 11:01 AM

Practice Homework Problems

I will list some typical problems involving universal gravity and gravitational potential energy here and will work out the solutions below.

Use the following constants for these problems:

G (universal gravitational constant) = 6.67×10^{-11} N m^2 / kg^2

$m_{earth} = 6 \times 10^{24}$ kg

$R_{earth} = 6.4 \times 10^{6}$ m

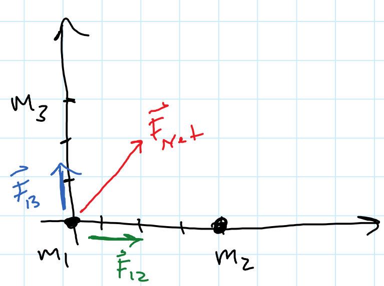
$m_{moon} = 7.4 \times 10^{22}$ kg

$R_{moon} = 1.7 \times 10^{6}$ m

1) Three small masses are located at the following locations:
   m_1 = 10 kg at (0, 0)
   m_2 = 20 kg at (4, 0)
   m_3 = 30 kg at (0, 3)
   Find the net gravitational force on m_1, due to the other two masses.
2) Find the free fall acceleration on the surface of the moon.
3) A satellite is in circular orbit around the earth at an altitude of 2×10^6 m above the earth's surface ($m_{satellite}$ = 100 kg).
   a. Find the gravitational force on the satellite due to the earth.
   b. Find the potential energy of the satellite / earth system.
4) If we ignore air resistance and the rotation of the earth, to what altitude will a projectile rise if launched straight up from the surface of the earth with a speed of 10,000 m/s?
5) If you launch a projectile straight up from the surface of the moon and you do not want it to return to the moon's surface, what launch speed is needed? This is called the escape speed, the speed a projectile needs to have when it leaves to never return.
6) If a meteor starts out at rest very far from the earth, what speed will it have when it gets to the top of earth's atmosphere (about 40,000 m above the surface of the earth)?

Solutions:

1)



Force between m_1 and m_3:

$$F_{13} = \frac{G m_1 m_3}{r_{13}^2} = \frac{(6.67 \times 10^{-11})(10)(30)}{3^2}$$

$$= 2.22 \times 10^{-9} \text{ N} \quad [\text{up} \uparrow]$$

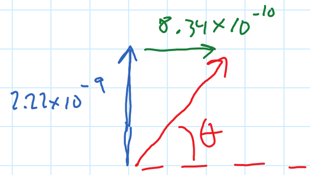
Force between m_1 and m_2:

$$F_{12} = \frac{G m_1 m_2}{r_{12}^2} = \frac{(6.67 \times 10^{-11})(10)(20)}{4^2}$$

$$= 8.34 \times 10^{-10} \text{ N} \quad [\text{right} \rightarrow]$$

Net force is the vector sum of those 2 forces:

$\vec{F}_{net} = \vec{F}_{12} + \vec{F}_{13}$



$$F_{net} = \sqrt{(2.22\times10^{-9})^2 + (8.34\times10^{-10})^2} = 2.37\times10^{-9}\ N$$

$$\theta = \tan^{-1}\left(\frac{2.22\times10^{-9}}{8.34\times10^{-10}}\right) = 69.4^\circ$$

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2) since $F = mg$ and $F = \dfrac{GmM}{R^2}$

$g = \dfrac{GM}{R^2}$ on surface of a planet or moon

$$g_{moon} = \frac{G\,M_{moon}}{R_{moon}^2} = \frac{(6.67\times10^{-11})(7.4\times10^{22})}{(1.7\times10^{6})^2}$$

$$= 1.71\ \frac{m}{s^2}$$ (about $\frac{1}{6}\,g_{earth}$)

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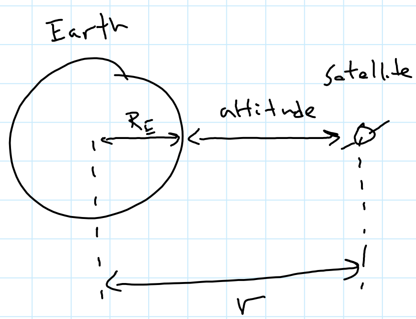
3) a) $F = \dfrac{G\,m_s M_E}{r^2}$



$r = R_E + \text{Altitude}$

$= 6.4\times10^{6}\ m + 2\times10^{6}\ m$

$= 8.4\times10^{6}\ m$

$$F = \frac{(6.67\times10^{-11})(100)(6\times10^{24})}{(8.4\times10^{6})^2} = 567\ N$$

b) $$U_g = -\frac{G m_s M_E}{r} = -\frac{(6.67\times10^{-11})(100)(6\times10^{24})}{(8.4\times10^{6})}$$

$$= -4.76\times10^{9}\ \text{J}$$

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4) Use energy. But, since V_i is so large, we will assume we cannot use $U_g = mgy$ (only good near surface of the Earth) and we will use $U_g = -\frac{G m_1 m_2}{r}$

$$E_i = E_f \quad \text{(ignoring air resistance)}$$

$$K_i + (U_g)_i = K_f + (U_g)_f$$

Cannot set one of these to zero. U_g can only be zero at r = infinity

$$\frac{1}{2}\cancel{m}V_i^2 - \frac{G\cancel{m}M_E}{r_i} = \frac{1}{2}\cancel{m}V_f^2 - \frac{G\cancel{m}M_E}{r_f}$$

Don't Forget Negatives

m = mass of projectile

$r_i = R_E$ (launching from surface of Earth)

$V_i = 10{,}000\ \frac{m}{s}$ (given)

$V_f = 0$ (max height)

solve for r_f

$$\frac{1}{2}V_i^2 - \frac{G M_E}{R_E} = -\frac{G M_E}{r_f}$$

$$\frac{1}{2}(10{,}000)^2 - \frac{(6.67\times10^{-11})(6\times10^{24})}{6.4\times10^{6}} = \frac{-(6.67\times10^{-11})(6\times10^{24})}{r_f}$$

$$r_f = 3.2\times10^{7}\ \text{m} \quad \text{from center of Earth}$$

altitude $= r_f - R_E = 2.55\times10^{7}$ m

5) Just like the previous problem:

$$E_i = E_f$$

$$K_i + (U_g)_i = K_f + (U_g)_f$$

$$\frac{1}{2} m v_i^2 - \frac{G m M_m}{r_i} = \frac{1}{2} m v_f^2 - \frac{G m M_m}{r_f}$$

In order for it Not to return it must just make it to r = infinity. In other words, its speed goes to zero just as its distance goes to infinity.

$$v_f \to 0$$

$$r_f \to \infty$$

$$\frac{1}{2} m v_i^2 - \frac{G m M_m}{R_m} = 0 - 0$$

$$\frac{1}{2} \cancel{m} v_i^2 = \frac{G \cancel{m} M_m}{R_m}$$

$$v_i = \sqrt{\frac{2 (6.67\times10^{-11})(7.4\times10^{22})}{1.7\times10^{6}}}$$

$$= 2{,}410 \ \frac{m}{s}$$

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6) let's use energy:

$$E_i = E_f$$

$$K_i + (U_g)_i = K_f + (U_g)_f$$

$$\frac{1}{2} m v_i^2 - \frac{G m M_E}{r_i} = \frac{1}{2} m v_f^2 - \frac{G m M_E}{r_f}$$

Meteor starts from rest $v_i = 0$

at $r_i = \infty$

$$0 - 0 = \frac{1}{2} \cancel{m} v_f^2 - \frac{G \cancel{m} M_E}{r_f}$$

$$\frac{1}{2} v_f^2 = \frac{G M_E}{r_f}$$

$$r_f = R_E + 40{,}000 \text{ m}$$
$$= 6.4 \times 10^6 + 40{,}000$$
$$= 6.44 \times 10^6 \text{ m}$$

$$v_f = \sqrt{\frac{2\left(6.67 \times 10^{-11}\right)\left(6 \times 10^{24}\right)}{6.44 \times 10^6}}$$

$$= 11{,}148 \ \frac{m}{s}$$