Phy 2A 6/14

Wednesday, June 14, 2017 9:57 AM

Goals for the Lecture:

1) Know how to calculate the angular momentum of objects moving in a straight lines
2) Know how to calculate the angular momentum of rotating rigid objects
3) Be able to use conservation of angular momentum to solve rotational collision problems

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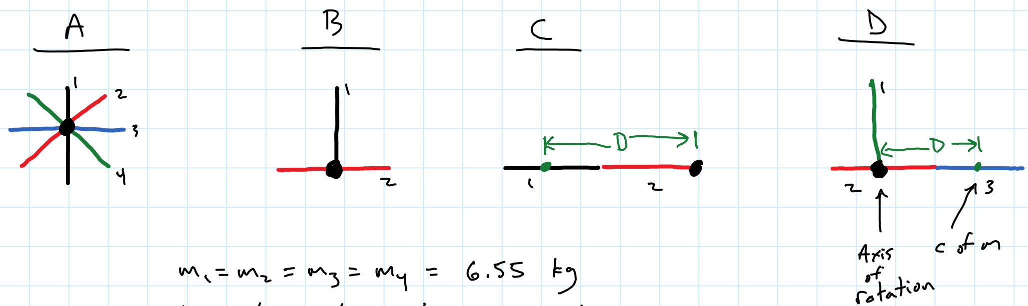
Pre-Lecture #3



$m_1 = m_2 = m_3 = m_4 = 6.55\ kg$

$L_1 = L_2 = L_3 = L_4 = 0.241\ m$

$I_{\text{rod rotating about center}} = \frac{1}{12} m L^2$

$I_{\text{rod rotating about end pt}} = \frac{1}{3} m L^2$

$I_A = I_1 + I_2 + I_3 + I_4 = 4 I_1 = 4\left(\frac{1}{12} M L^2\right) = 0.127\ kg\ m^2$

$I_B = I_1 + I_2 = \frac{1}{3} m_1 L_1^2 + \frac{1}{12} m_2 L_2^2 = 0.159\ kg\ m^2$

$I_c \rightarrow$ Like one long rod $\quad m_{total} = 2m \quad L_{total} = 2L$

$I_c = \frac{1}{3} m_{total} L_{total}^2$

$= \frac{1}{3}(13.1)(0.482)^2 = 1.01\ kg\ m^2$

Like 2 rods

$I_c = I_1 + I_2 = \underbrace{I_{cm} + m_1 D^2}_{I_1} + I_2$

$= \underbrace{\frac{1}{12} m_1 L_1^2 + m_1\left(\frac{3L}{2}\right)^2}_{I_1} + \underbrace{\frac{1}{3} m_2 L_2^2}_{I_2}$

$= \left(\frac{1}{12} + \frac{9}{4} + \frac{1}{3}\right) m L^2 = 1.01\ kg\ m^2$

$I_D = I_1 + I_2 + I_3 = \frac{1}{3} m_1 L_1^2 + \frac{1}{12} m_2 L_2^2 + \underbrace{I_{cm} + m_3 D^2}_{I_3}$

$$= \frac{1}{3} m_1 L_1^2 + \frac{1}{12} m_2 L_2^2 + \underbrace{\frac{1}{12} m_3 L_3^2 + m_3 L^2}_{I_3}$$

$$= \left(\frac{1}{3} + \frac{1}{12} + \frac{13}{12} \right) mL^2 = 0.571 \text{ kg m}^2$$

Angular Momentum: $\vec{L}$

$$L = \begin{cases} I\omega & \text{object that is rotating} \\ mv\,d_\perp & \text{object moving in a straight line} \end{cases}$$

$d_\perp$: perpendicular distance from line of velocity to the axis of rotation

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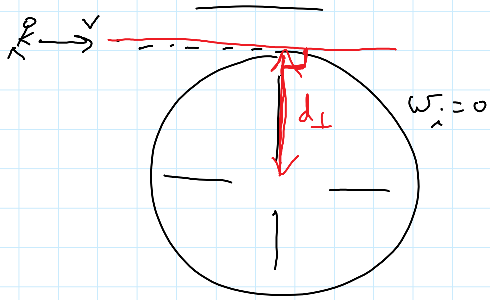
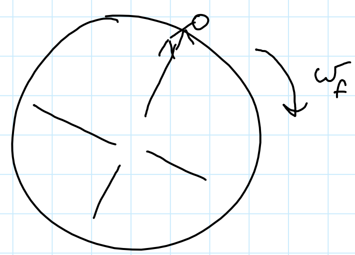
Example:

initial



final



$L \neq 0$

Rotating → must have angular momentum

So, we must have angular momentum initially

$L \neq 0$

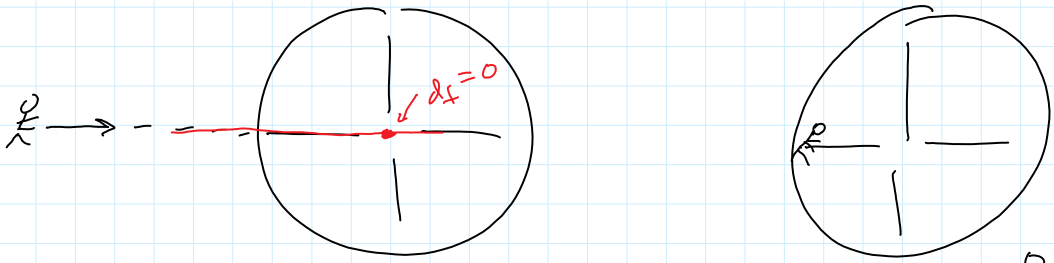
So, the kid must have angular momentum

____ ____ ____ ____ ____

OR

initial

final



$\omega_i = 0$ $\qquad$ $\omega_f = 0$

$L_f = 0$

so

$L_i = 0$

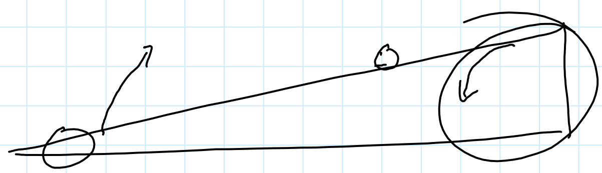
Angular Momentum:

$$\vec{L}_i = \vec{L}_f \qquad \circlearrowleft +$$

ice skater spinning with arms out $\Rightarrow$ ice skater spinning with arms in

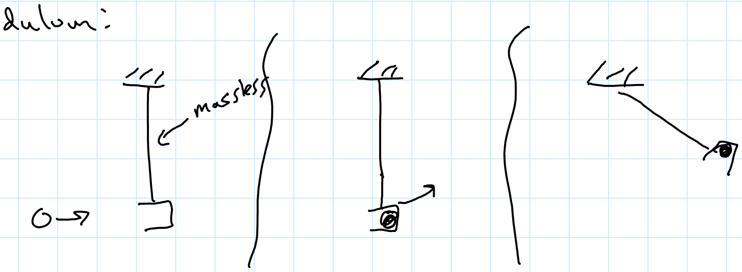
$$I_i \, \omega_i = I_f \, \omega_f$$

Application:
Drag race car



Large I prevents car from flipping over when it tries to conserve angular momentum

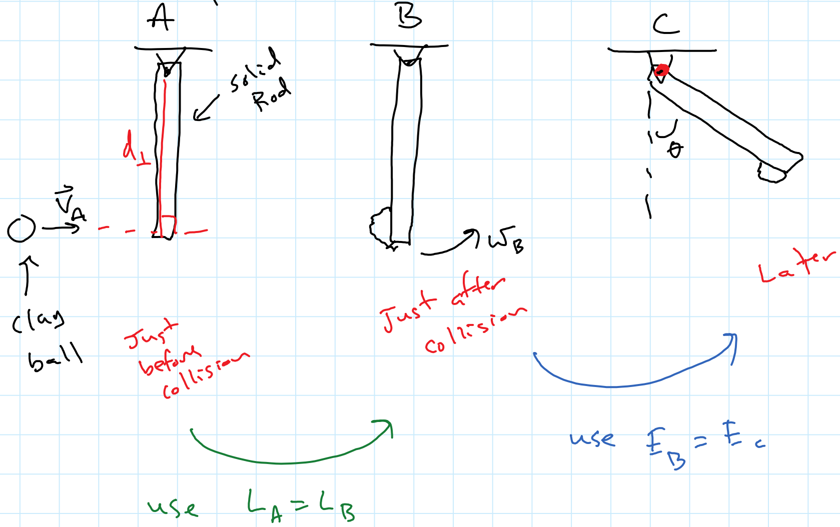
Ballistic Pendulum:



used Linear Momentum because we

assumed the rod was massless

For this problem:



we need to use angular momentum

given: clay ball: $m_c = 0.6$ kg

$V_A = 5\ \frac{m}{s}$

$r_{clay} << l$ (Like a point mass)

Rod: solid, uniform rod

$l = 1.2$ m

$m_R = 0.3$ kg

find: θ (max angle)

$$L_A = L_B \quad \circlearrowleft +$$

$$(L_A)_{clay} + \cancel{(L_A)_{rod}}^{\,0} = (L_B)_{clay + rod}$$

$$m_c V_A d_\perp = (L_B)_{clay + rod}$$

$$m_c V_A l = I_{total}\, \omega_B$$

$$m_c v_A l = (I_{clay} + I_{rod})\, \omega_B$$

$$m_c V_A l = \left(m_c l^2 + \frac{1}{3} m_R l^2\right) \omega_B$$

$$(0.6)(5)(1.2) = \underbrace{\left[(0.6)(1.2)^2 + \frac{1}{3}(0.3)(1.2)^2\right]}_{1.008} \omega_B$$

1.008

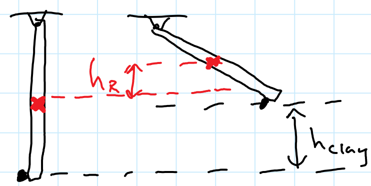
$$\omega_B = 3.57 \ \frac{rad}{s}$$

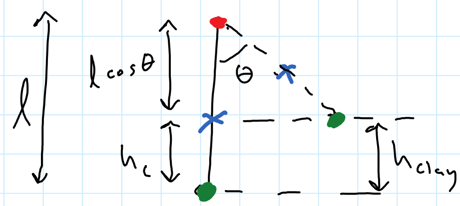
Now, use energy to go from time B to time C

$$E_B = E_c$$ No friction or external forces

$$\left(K_{rotational}\right)_B = \left(U_g\right)_c$$

$$\frac{1}{2} I_{both} \omega_B^2 = m_c g h_{clay} + m_{rod} g h_R$$





$$l\cos\theta + h_c = l$$

$$h_c = l - l\cos\theta$$

$$h_R = \frac{l}{2} - \frac{l}{2}\cos\theta$$

$$h_R = \frac{1}{2} h_{clay}$$

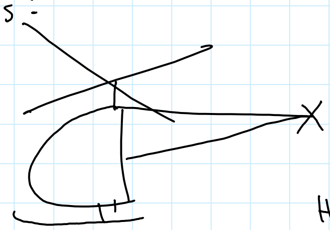
$$\frac{1}{2} I_{both} \omega_B^2 = m_c g h_{clay} + m_{rod} g \frac{h_{clay}}{2}$$

$$\frac{1}{2}(1.008)(3.57)^2 = (0.6)(9.8)h_{clay} + (0.3)(9.8)\frac{h_{clay}}{2}$$

$$h_{clay} = 0.245 \ m = l - l\cos\theta$$

$\theta = 37.3°$

Applications:



← keeps helicopter from spinning around

Helicopter would spin to conserve angular momentum

Spinning projectiles to get them to travel straight

- Football (American football)
- bullets

Problem: kid runs and jumps onto merry-go-round

find: ω_f

given: $m_{kid} = 40$ kg

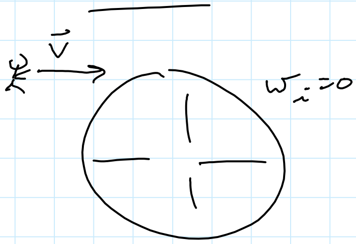
$v = 3 \frac{m}{s}$

$m_{mgR} = 100$ kg

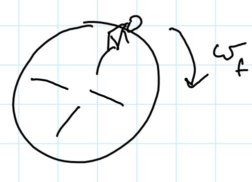
$R = 2$ m

shape: solid disk

initial



final



$$L_i = L_f \quad \curvearrowright +$$

$$(L_{kid})_i + (L_{mgR})_i = (L_{\text{both together}})_f$$

$$m_{kid} v d_\perp + 0 = I_{both} \omega_f$$

$$m_{kid} V d_{\perp} + 0 = I_{both}\, \omega_f$$

$$m_{kid} V R = (I_{kid} + I_{mgr})\, \omega_f$$

$$m_{kid} V R = \left[m_{kid} R^2 + \frac{1}{2} M_{mgr} R^2 \right] \omega_f$$

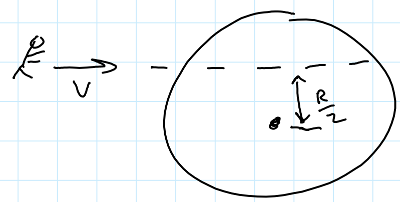
$$(40)(3)(2) = \left[(40)(2)^2 + \frac{1}{2}(100)(2)^2 \right] \omega_f$$

$$240 = (160 + 200)\, \omega_f$$

$$\omega_f = \frac{2}{3} \frac{rad}{s}$$

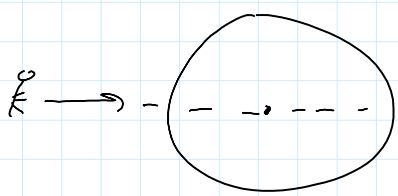
Find angular momentum of the kid about the axis of rotation at the center of the merry-go-round

a)



$$L_{kid} = m_{kid} V \frac{R}{2}$$

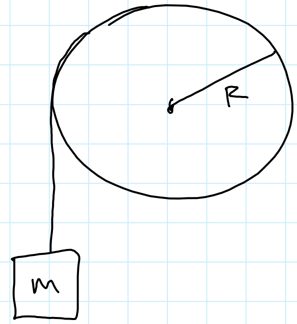
b)



$$L_{kid} = 0$$

since $d_{\perp} = 0$

Problem:



given: hanging mass: $m = 5$ kg

Pulley mass: $M_p = 4$ kg
solid disk

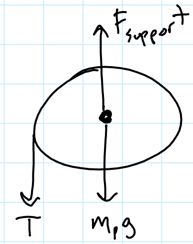
$R = 0.3$ m

Find: 1) tension in Rope
2) acceleration of hanging mass

FBD

hanging mass

pulley



$$\Sigma F = ma \quad \downarrow +$$

$$mg - \underline{T} = m\underline{a}$$

$$\Sigma \tau = I\alpha \quad \circlearrowleft +$$

$$\tau_T + \cancelto{0}{\tau_{support}} + \cancelto{0}{\tau_{mg}} = I\alpha$$

$$\underline{T}R = \left(\tfrac{1}{2} m_p R^2\right) \underline{\alpha}$$

$$a = R\alpha$$

$$TR = \left(\tfrac{1}{2} m_p R^2\right) \frac{a}{R}$$

$$T = \frac{m_p}{2} a$$

$$mg - \frac{m_p}{2} a = ma$$

$$mg = \left(m + \frac{m_p}{2}\right) a$$

$$5(9.8) = \left(5 + \frac{4}{2}\right) a$$

$$a = 7 \ \frac{m}{s^2}$$

$$T = \frac{m_p}{2} a = 14 \ N$$