Phy 2A 6/12

Monday, June 12, 2017 9:54 AM

Goals for the Lecture:

1) Understand how to use energy with a rolling object
2) Be able to use energy to solve problems that include rotational motion

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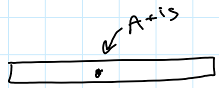
Rotational Inertia:

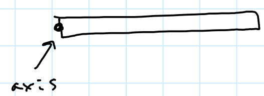
Point object: $I = mr^2$

Rod (mid-point):

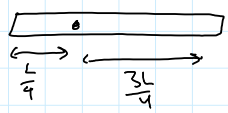


$I = \frac{1}{12} ML^2$

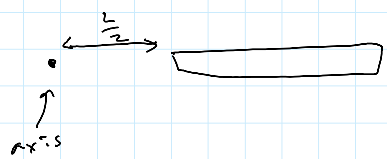
Rod (end-point):



$I = \frac{1}{3} ML^2$



$I = \frac{7}{48} ML^2$

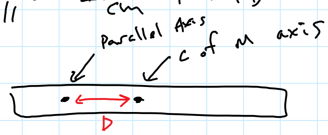


$I = \frac{13}{12} ML^2$

see below

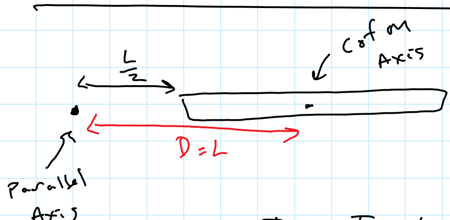
Parallel Axis Theorem:

$$I_{\parallel} = I_{cm} + MD^2$$



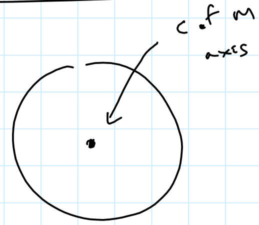
$I_{cm} = \frac{1}{12} ML^2$

$$I_{\parallel} = I_{cm} + MD^2$$
$$= \frac{1}{12} ML^2 + M\left(\frac{L}{4}\right)^2$$
$$= \left(\frac{1}{12} + \frac{1}{16}\right) ML^2$$
$$= \frac{7}{48} ML^2$$

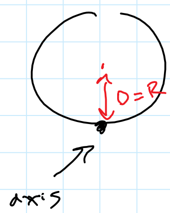


$$I_{\parallel} = I_{cm} + MD^2$$
$$= \frac{1}{12} ML^2 + ML^2$$

$$= \frac{13}{12} ML^2$$



$$I_{cm} = \frac{1}{2} MR^2$$

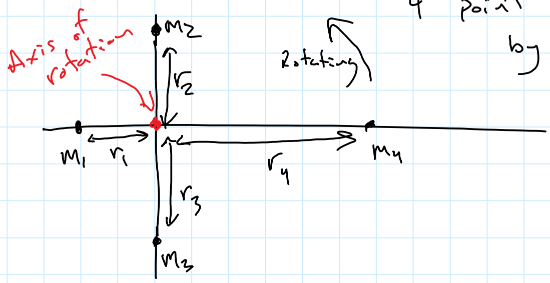


$$I = I_{cm} + MD^2$$

$$= \frac{1}{2}MR^2 + MR^2$$

$$= \frac{3}{2} MR^2$$

Rotational KE :



4 point masses connected by massless rods

Find KE:

$$K = K_1 + K_2 + K_3 + K_4$$

$$= \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 + \frac{1}{2}m_3V_3^2 + \frac{1}{2}m_4V_4^2$$

using $v = r\omega$

$$= \frac{1}{2}m_1(r_1\omega)^2 + \frac{1}{2}m_2(r_2\omega)^2 + \frac{1}{2}m_3(r_3\omega)^2 + \frac{1}{2}m_4(r_4\omega)^2$$

$$= \frac{1}{2}(m_1r_1^2)\omega^2 + \frac{1}{2}(m_2r_2^2)\omega^2 + \frac{1}{2}(m_3r_3^2)\omega^2 + \frac{1}{2}(m_4r_4^2)\omega^2$$

$$= \frac{1}{2}I_1\omega^2 + \frac{1}{2}I_2\omega^2 + \frac{1}{2}I_3\omega^2 + \frac{1}{2}I_4\omega^2$$

$$= \frac{1}{2}(I_1 + I_2 + I_3 + I_4)\omega^2$$

$$= \frac{1}{2} I \omega^2$$

I total for object

the rotating obj

$$\boxed{K_R = \frac{1}{2} I \omega^2}$$ Rotational Kinetic Energy

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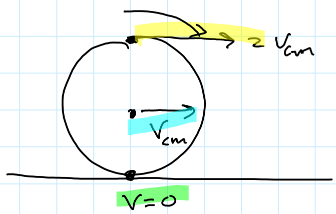
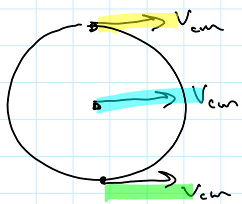
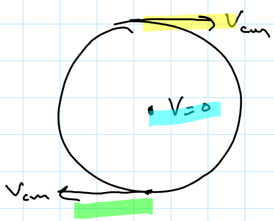
Rolling:



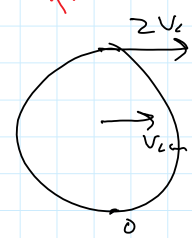
pure translational motion (No rotation):



Pure rotational motion (No translation):

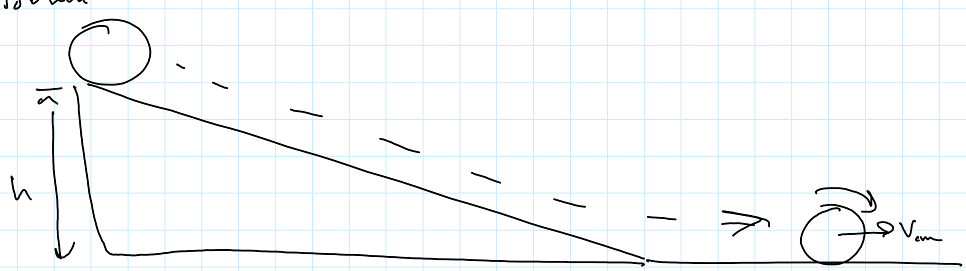


Combine these



you get Rolling

Energy Problem:



given:
- Released from rest
- object is a solid disk
- $h = 2$ m
- $m = 1.5$ kg
- radius $= 0.3$ m

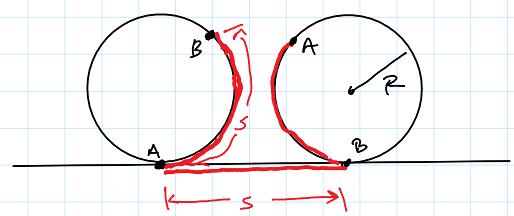
find: V_{cm} (after Rolling down the hill)

$$E_i + W_{friction} + W_{ext} = E_f$$

$$(U_g)_i + 0 + 0 = (K + K_R)_f$$

$$mgh = \frac{1}{2} m V_{cm}^2 + \frac{1}{2} I \omega^2$$

how are V_{cm} and ω related when an object is rolling?



$$V_{cm} = R\omega$$

$$mgh = \frac{1}{2} m V_{cm}^2 + \frac{1}{2} I \left(\frac{V_{cm}}{R}\right)^2$$

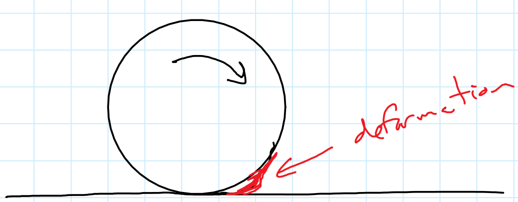
$$\cancel{m}gh = \frac{1}{2} \cancel{m} V_{cm}^2 + \frac{1}{2}\left(\frac{1}{2}\cancel{M}\cancel{R^2}\right)\frac{V_{cm}^2}{\cancel{R^2}}$$

$$gh = \left(\frac{1}{2} + \frac{1}{4}\right) V_{cm}^2$$

$$\sqrt{\frac{4}{3} gh} = V_{cm}$$

$$V_{cm} = 5.11 \ \frac{m}{s}$$

Rolling friction

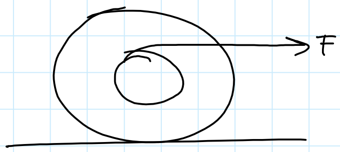


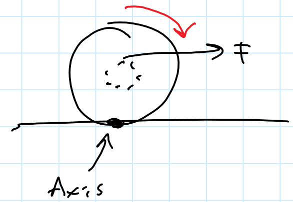
if objects are perfectly Non-deformable (Like in our class)

$\rightarrow$ No Rolling friction

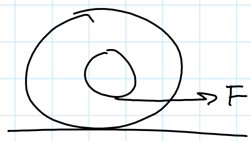
Torque demo:

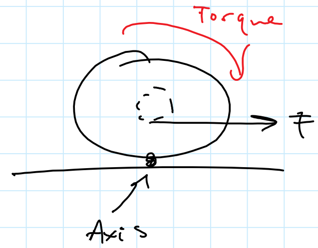
which way does it rotate?



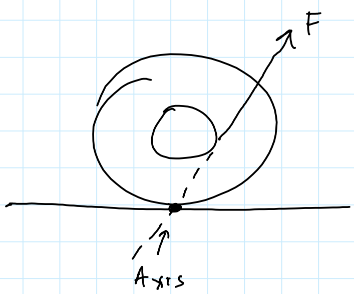


clockwise Rotation





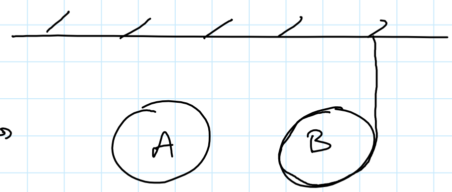
clockwise rotation

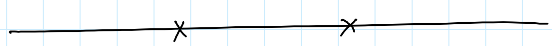


No torque → No rotation

Demo:

Release from Rest →





a) where do they hit the ground?

A → on the "X"

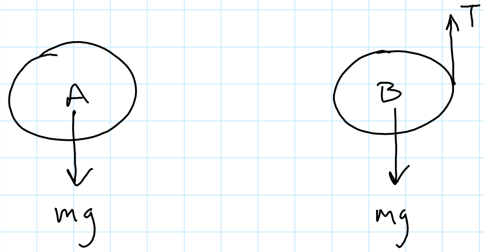
B → on the "X"

on the "X"
to the right
to the left

b) Which hits the ground first?

A hits first

Draw a free body diagram for each:



$$\Sigma F = ma$$

$$\Sigma F_A = mg \qquad \Sigma F_B = mg - T$$

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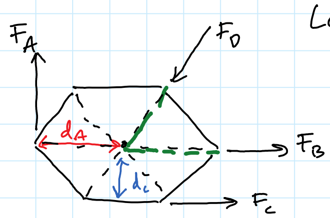
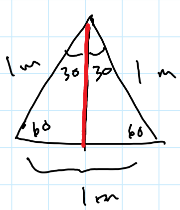
worksheet p. 211

Top: Find the torque produced by each Force and the Net torque

Let ccw be + direction

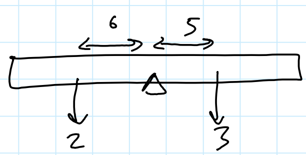


	Force	lever arm (d)	Torque (F×d)
A	4 N	1 m	− 4 N m
B	4 N	0	0
C	4 N	0.866	+ 3.46 N m
D	6 N	0	0



Net $\tau_{Net} = -0.54$ Nm

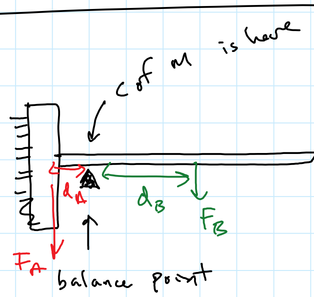
bottom:



$$\left.\begin{aligned} \tau_{ccw} &= (2)(6) = 12 \\ \tau_{cw} &= (3)(5) = 15 \end{aligned}\right\} \quad \tau_{cw} > \tau_{ccw}$$

it will rotate cw

Broom demo:



cut at c of m → which end is heavier?

Broom end is heavier than handle end because it produces the same torque with a smaller lever arm.

$$\tau_{\circlearrowleft} = \tau_{\circlearrowright}$$

$$F_A d_A = F_B d_B$$

since $d_A < d_B$

F_A must be greater than F_B

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worksheet p. 216

Top: Find Torque:

hanging mass

$$\tau = F d = mgR$$

$$\tau_A = (0.5)(9.8)(0.1\ m) = 0.49\ N\ m$$

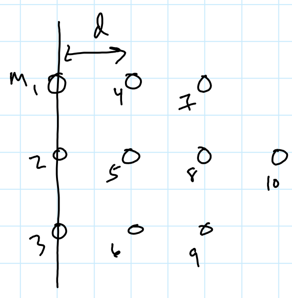
$$\tau_B = (0.2)(9.8)(0.05\ m) = 0.098\ N\ m$$

$$\tau_c = (0.5\ kg)(9.8)(0.05\ m) = 0.245\ N\ m$$

$$\tau_D = (0.8\ kg)(9.8)(0.1\ m) = 0.784$$

bottom: $I_c > I_A > I_D > I_B$

For A)



$$I = I_1 + I_2 + I_3 + \dots + I_{10}$$

$$I_1 = I_2 = I_3 = m(0) = 0$$

$$I_4 = I_5 = I_6 = m d^2$$

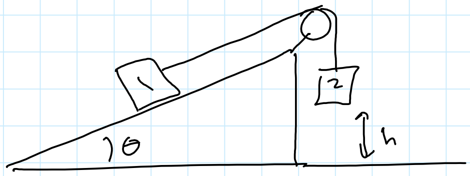
$$I_7 = I_8 = I_9 = m(2d)^2 = 4md^2$$

$$I_{10} = m(3d)^2 = 9md^2$$

$$I = 24\ md^2$$

$I = 24 \text{ md}$

problem:



given: starts from rest

$m_1 = 10$ kg

$m_2 = 20$ kg

$m_p = 1.4$ kg

$R_p = 0.2$ m

$\theta = 30°$

$\mu_K = 0.15$

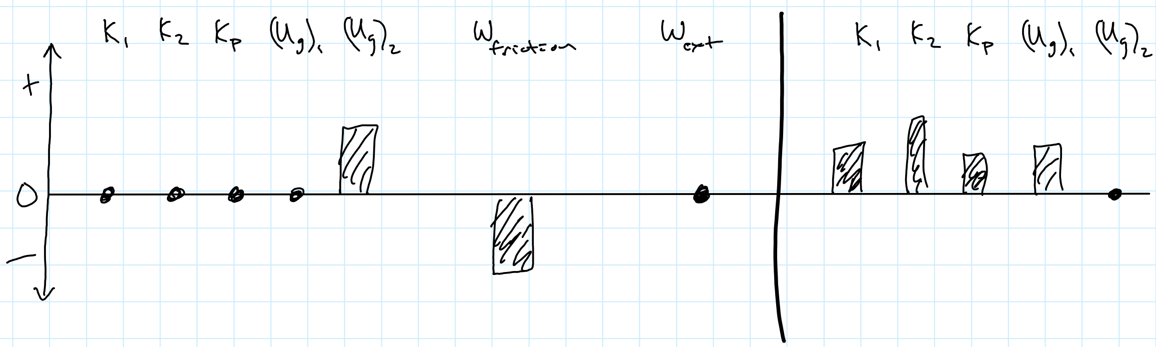
$h = 1.8$ m

pulley is a solid disk

find: V_f (velocity of m_2 just before it hits the ground)

use energy:

$$E_i + W_{friction} + W_{ext} = E_f$$



$$m_2 g h - \mu_K (m_1 g \cos\theta) h = \frac{1}{2} m_1 V_f^2 + \frac{1}{2} m_2 V_f^2 + \frac{1}{2} I \omega_f^2 + m_1 g h \sin\theta$$

$$V_f = R_p \omega_f$$

$$I = \frac{1}{2} M_p R_p^2$$

$$m_2 g h - \mu_K m_1 g \cos\theta \, h = \frac{1}{2} m_1 V_f^2 + \frac{1}{2} m_2 V_f^2 + \frac{1}{2}\left(\frac{1}{2} m_p \cancel{R_p^2}\right)\left(\frac{V_f}{\cancel{R_p}}\right)^2 + m_1 g h \sin\theta$$

$$m_2 g h - \mu_K m_1 g \cos\theta \, h - m_1 g h \sin\theta = \frac{1}{2}\left(m_1 + m_2 + \frac{m_p}{2}\right) V_f^2$$

$$(20)(9.8)(1.8) - (0.15)(10)(9.8)\cos 30° (1.8) - (10)(9.8)(1.8)\sin 30° = \frac{1}{2}\left(10 + 20 + \frac{1.4}{2}\right) V_f^2$$

$$352.8 - 22.9 - 88.2 = 15.35 \, V_f^2$$

$$241.7 = 15.35 \; v_f^2$$

$$v_f = 3.97 \; \frac{m}{s}$$