Goals for the Lecture:

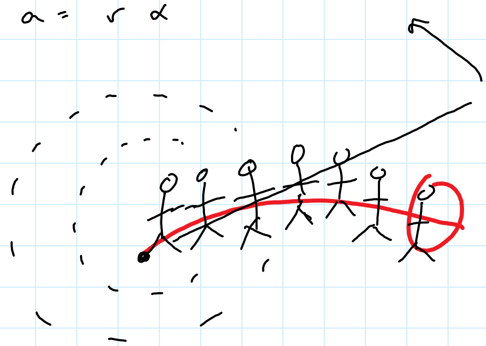
1) Understand how to use rotational kinematics equations to solve rotation problems
2) Understand what torque is and how to calculate it
3) Understand how to use Newton's 2nd Law for rotation ($\sum \tau = I\alpha$) to solve problems
4) Know how to use the chart of rotational inertias to find rotational inertia of common shapes about typical axes of rotation

if we use radians:

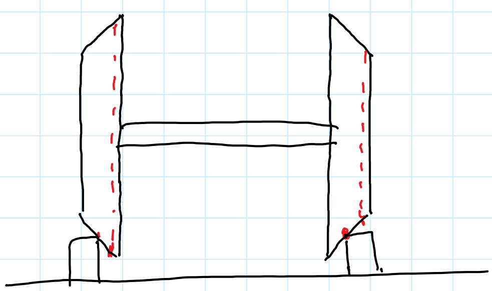
$$s = r\theta$$

$$v = r\omega$$

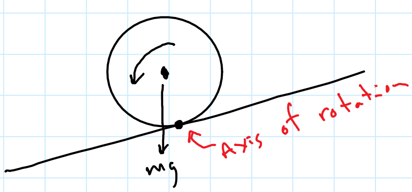
$$a = r\alpha$$

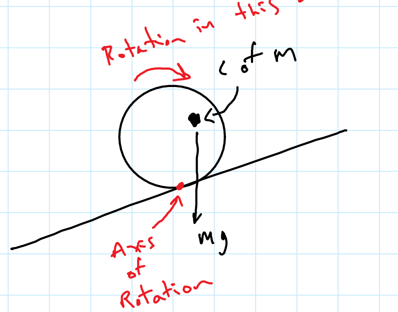


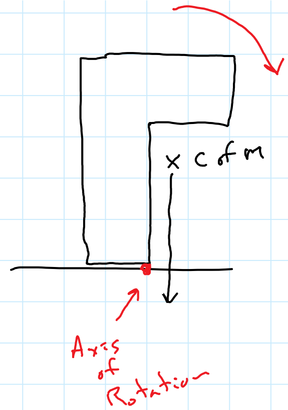
Train wheels:

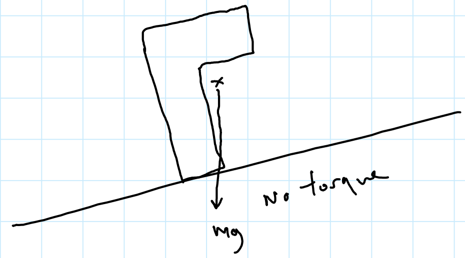


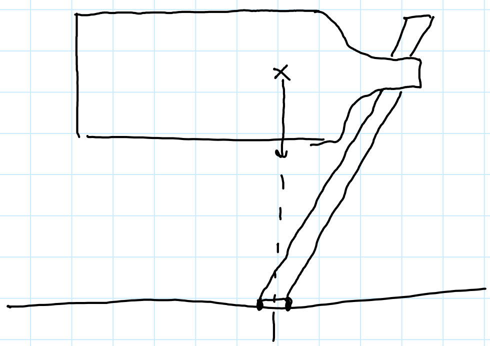
Torque : Torque causes objects to rotate





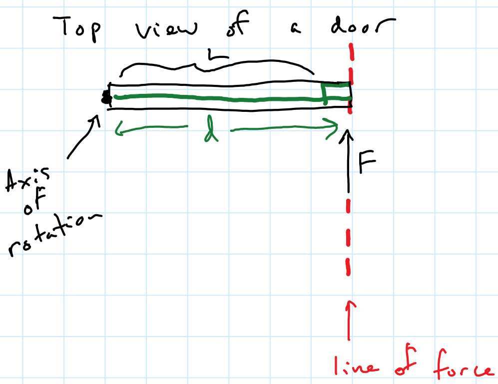






Torque:

1)



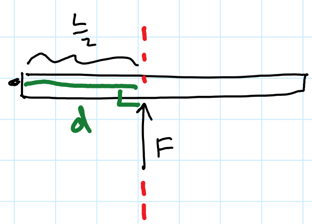
Torque: $\vec{\tau} = F\,d$

lever arm

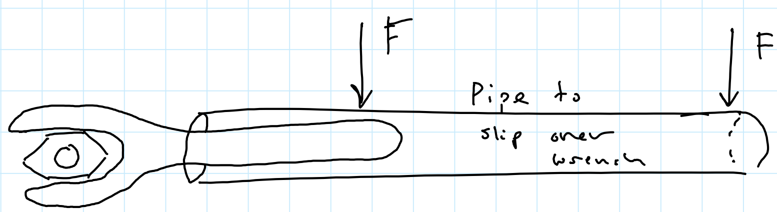
$\perp$ distance from the line of force to the axis of rotation

$$\tau = F L$$

2)

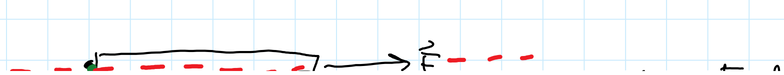


$$\tau = F\,d$$
$$= F\frac{L}{2}$$

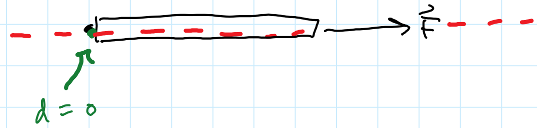


increases torque

3)

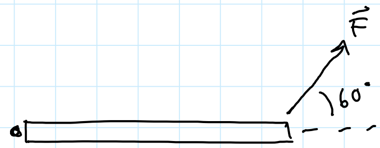


3)

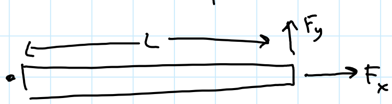


$$\tau = F\,d$$
$$= F(0)$$
$$= 0$$

4)

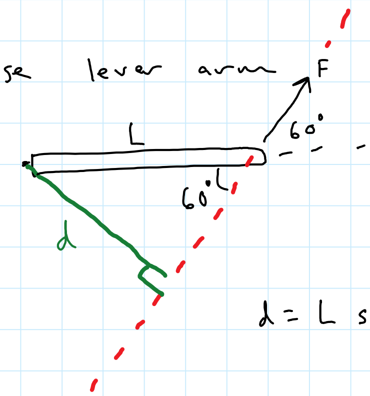


use components



$$\vec{\tau} = \vec{\tau}_{F_x} + \vec{\tau}_{F_y}$$
$$= 0 + F_y L$$
$$= F \sin 60^\circ\, L$$

use lever arm



$d = L \sin 60^\circ$

$$\tau = F\,d$$
$$= F L \sin 60^\circ$$

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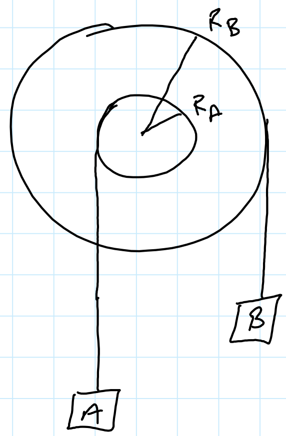
2 disks stuck together:



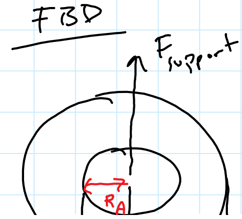
$M_A = M_B$

Which way does it rotate if released from rest?

Find Net torque:

$$\Sigma\vec{\tau} = \vec{\tau}_A + \vec{\tau}_{mg} + \vec{\tau}_{F_{support}} + \vec{\tau}_B \quad +$$

FBD

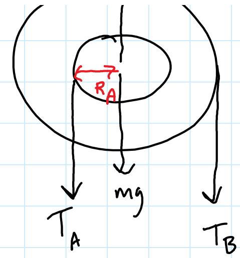


$$= T R + mg(0) + F (0) + T R$$

support

$$= -T_A R_A + mg(0) + F_{support}(0) + T_B R_B$$

Force   lever arm



$$= -T_A R_A + T_B R_B$$

$$= -(M_A g) R_A + (M_B g) R_B$$

since $M_A = M_B$

$R_A < R_B$

$\tau_{Net} > 0$ clockwise rotation!

Units: $\tau = Nm \neq J$ Work $= Nm = J$

not energy   energy

Joules for energy only

Equilibrium:

$$\sum \vec{F} = 0 \quad \text{and} \quad \sum \vec{\tau} = 0$$

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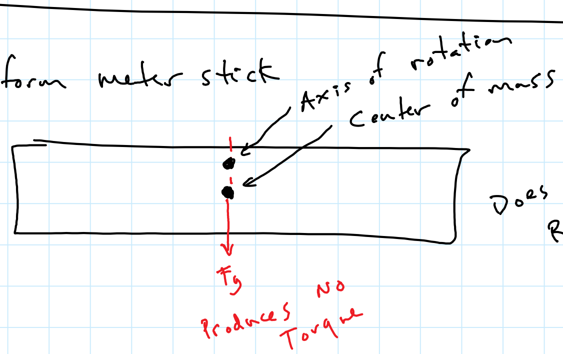
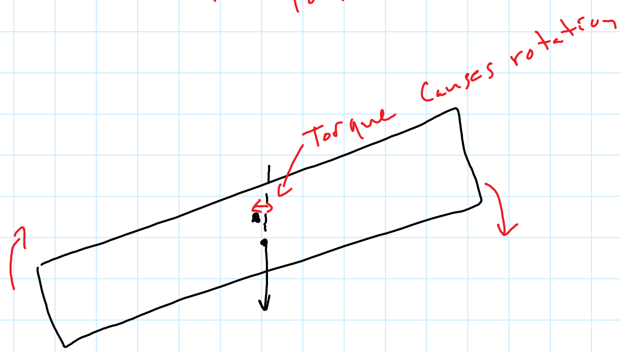
uniform meter stick



Does Not Rotate



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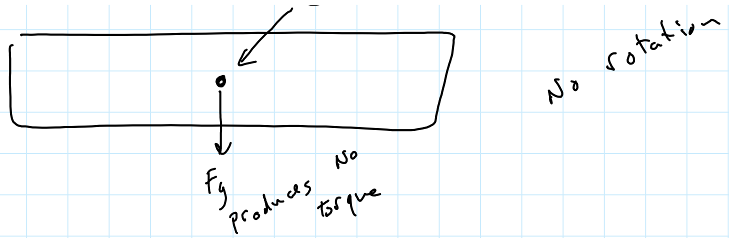
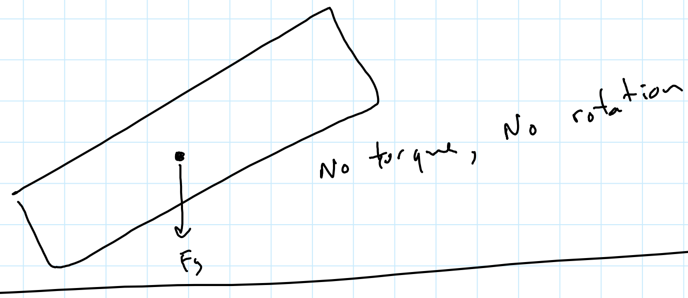
c of M and axis of rotation

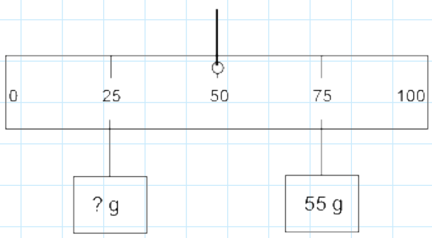


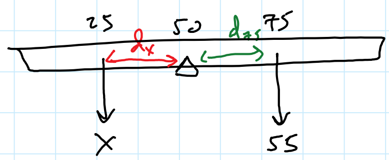


Torque Example - 1

A 145 g meter stick is suspended at the 50 cm mark.
If 55 g are added at the 75 cm mark, how many grams should be added at the 25 cm mark to keep the system in equilibrium?

1) 25 g
2) 55 g
3) 75 g
4) 145 g
5) 1495 g





$$\Sigma \vec{\tau} = 0 \qquad \circlearrowleft +$$

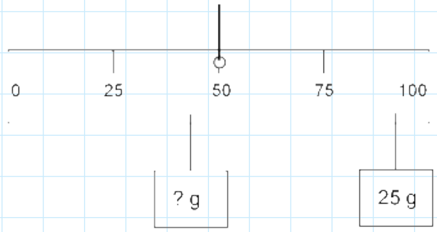
$$X(\cancel{9.8})(25\ \text{cm}) - 55(\cancel{9.8})(25\ \text{cm}) = 0$$

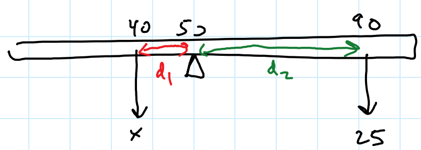
$$X = 55 \text{ grams}$$

Torque Example - 2

A 145 g meter stick is suspended at the 50 cm mark.
If 25 g are added at the 90 cm mark, how many grams should be added at the 40 cm mark to keep the system in equilibrium?

1) 25 g
2) 40 g
3) 90 g
4) 100 g
5) 1495 g





$$\Sigma \vec{\tau} = X d_1 - 25 d_2 = 0$$

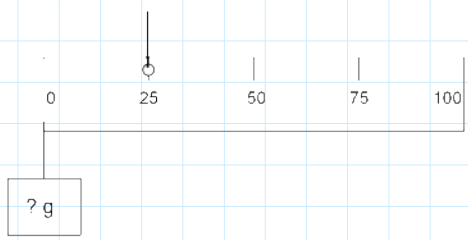
$$x(10) - 25(40) = 0$$

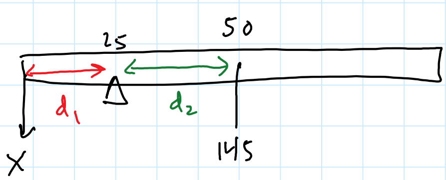
$$X = 100 \text{ grams}$$

Torque Example - 3

A 145 g meter stick is suspended at the 25 cm mark.
How many grams should be added at the zero cm mark to keep the system in equilibrium?

1) 25 g
2) 55 g
3) 90 g
4) 100 g
5) 145 g
6) 1495 g





massless meter stick with 145 g mass at the center of mass location

$$\Sigma \vec{\tau} = x d_1 - 145(d_2) = 0$$

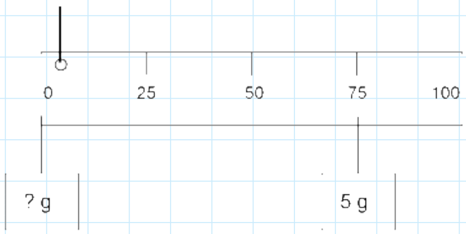
$$x(25) - 145(25) = 0$$

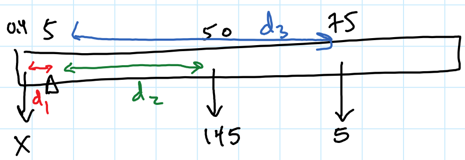
$$x = 145 \text{ grams}$$

Torque Example - 4

A 145 g meter stick is suspended at the 5 cm mark.
If 5 g are added at the 75 cm mark, how many grams should be added at the 0.4 cm mark to keep the system in equilibrium?

1) 25 g
2) 55 g
3) 90 g
4) 100 g
5) 145 g
6) 1495 g



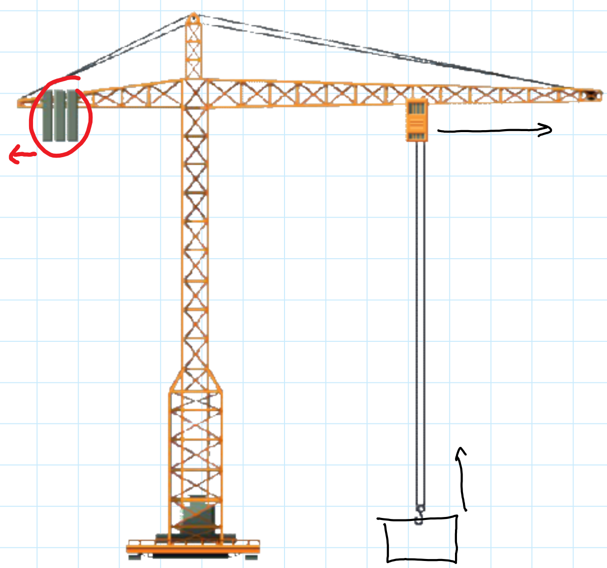


$$\Sigma \vec{\tau} = X d_1 - 145\, d_2 - 5\, d_3 = 0$$

$$X\underbrace{(5-0.4)}_{d_1} - 145\underbrace{(50-5)}_{d_2} - 5\underbrace{(75-5)}_{d_3} = 0$$

$X = 1495$ grams

Torque Application



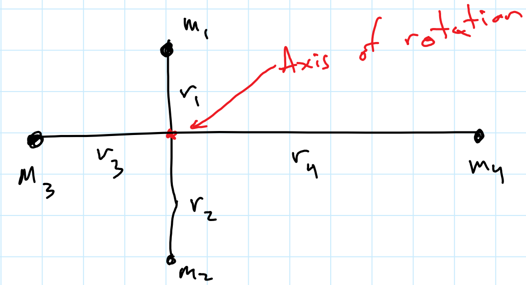
Rotational Inertia: I

point mass: $I = mr^2$

multiple point masses:

m_1 f rotation

multiple point masses:



$m_1 = 1$ kg

$m_2 = 2$ kg

$m_3 = 3$ kg

$m_4 = 4$ kg

$r_1 = 2$ m

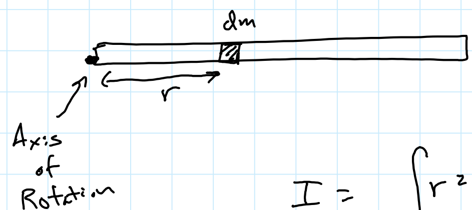
$r_2 = 3$ m

$r_3 = 4$ m

$r_4 = 6$ m

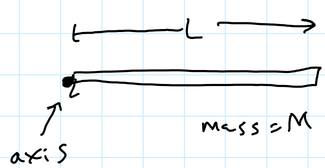
$$
\begin{aligned}
I &= I_1 + I_2 + I_3 + I_4 \\
&= m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2 \\
&= (1)(2)^2 + (2)(3)^2 + (3)(4)^2 + (4)(6)^2 \\
&= 4 + 18 + 48 + 144 \\
&= 214 \text{ kg m}^2
\end{aligned}
$$

Solid Objects: Like a rod



$$I = \int r^2 \, dm$$

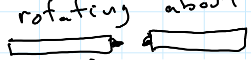
Look it up in the table in the book or formula sheet



$$I = \frac{1}{3} M L^2$$

$$I = \frac{1}{12} M L^2$$

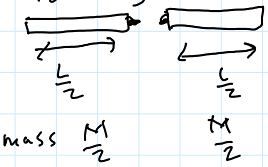
Like 2 rods
half the length & mass
rotating about their ends



$$I = I_1 + I_2$$

$$= \frac{1}{3} M' L'^2 + \frac{1}{3} M' L'^2$$

half the length & mass, rotating about their ends



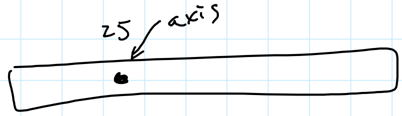
$$I = I_1 + I_2$$

$$= \frac{1}{3} M' L'^2 + \frac{1}{3} M' L'^2$$

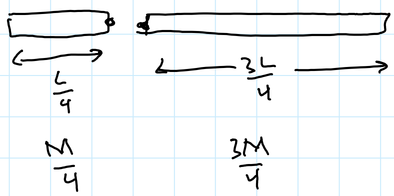
$$= \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 + \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2$$

$$= \frac{ML^2}{24} + \frac{ML^2}{24}$$

$$= \frac{1}{12} ML^2$$



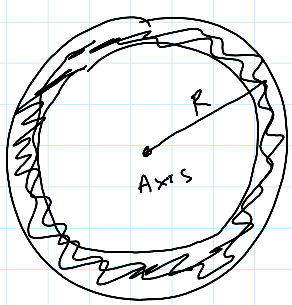
treat it as 2 rods



add up the I's

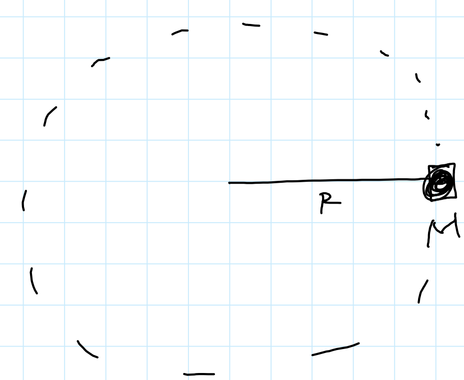
$$I = I_1 + I_2$$

Hollow hoop or ring:

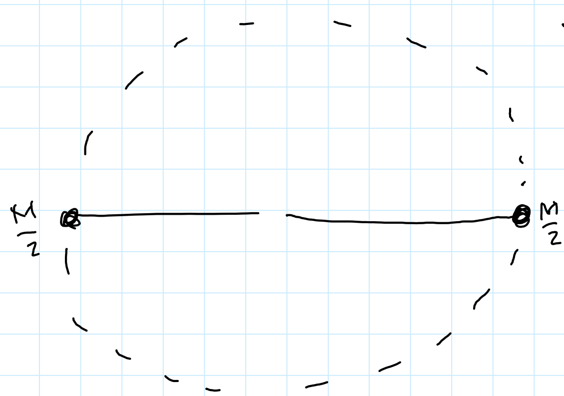


total mass M

$$I = MR^2$$

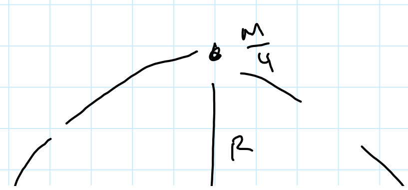


$$I = MR^2$$

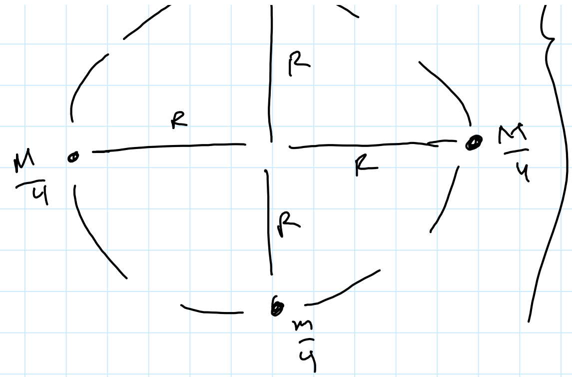


$$I = \frac{M}{2}R^2 + \frac{M}{2}R^2$$

$$= MR^2$$



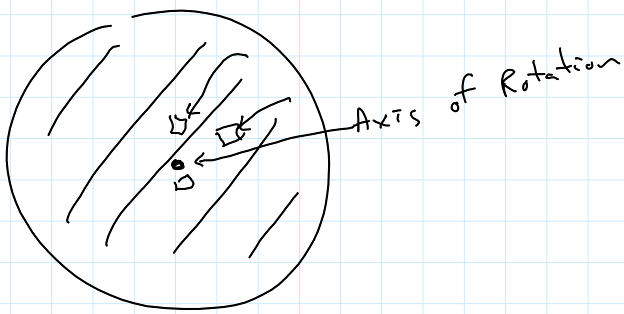
$$I = \frac{M}{4}R^2 + \frac{M}{4}R^2 +$$



$$I = \frac{M}{4}R^2 + \frac{M}{4}R^2 + \frac{m}{4}R^2 + \frac{M}{4}R^2$$

$$= MR^2$$

Solid disk



total mass M

radius R

$$I_{disk} < I_{hoop}$$

$$I_{disk} = \frac{1}{2} M R^2$$