

**Goals for the Lecture:**

- 1) Understand what Momentum is and why it is special (how is it similar to and different from kinetic energy?)
- 2) Understand what Impulse is (how is it similar to and different from work?)
- 3) Be able to use the Impulse – Momentum Theorem to solve problems
- 4) Use Conservation of Momentum to solve problems

$$\sum \vec{F} = m \vec{a}$$

$$= m \frac{d\vec{v}}{dt}$$

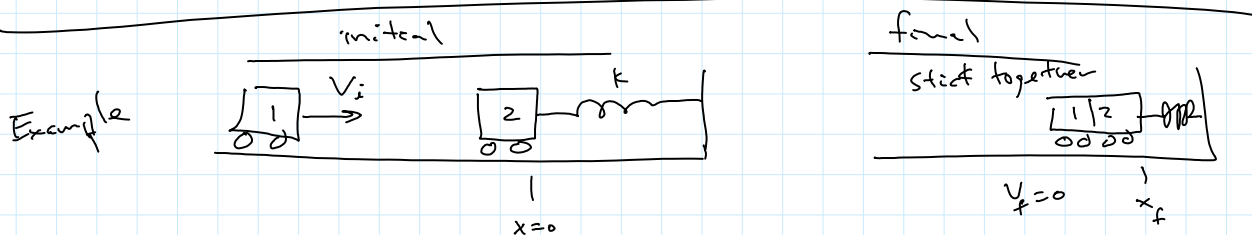
$$= \frac{d(m\vec{v})}{dt} \quad \text{if } m \text{ is constant}$$

$$\text{if } \sum \vec{F} = 0$$

$$\frac{d(m\vec{v})}{dt} = 0$$

$$m\vec{v} = \text{constant}$$

$$\text{momentum} \quad \vec{p} = m\vec{v}$$



given:  $m_1 = 10 \text{ kg}$

$m_2 = 20 \text{ kg}$

$v_i = 5 \frac{\text{m}}{\text{s}}$

$k = 50 \frac{\text{N}}{\text{m}}$

$v_f = 0$

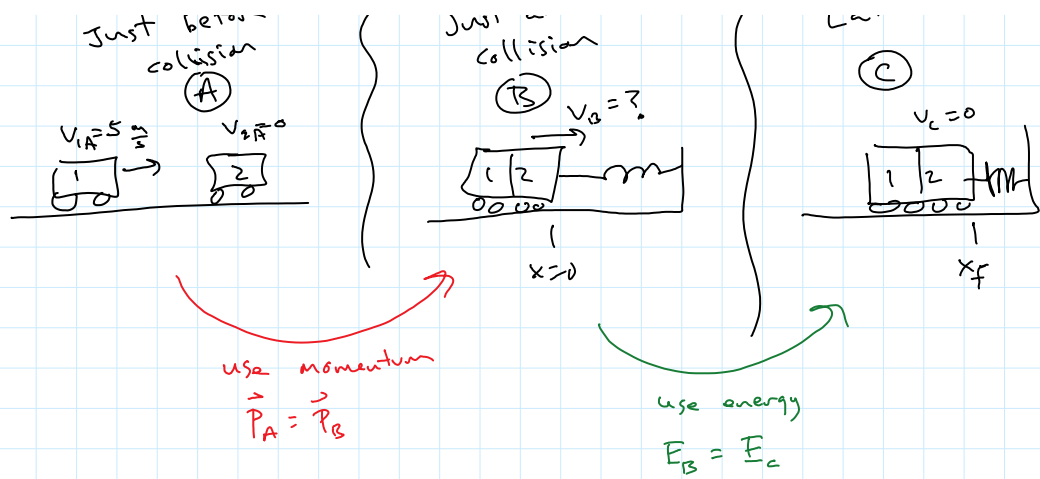
find:  $x_f$  (max compression of spring)

Timeline:

Just before  
collision  
(A)

Just after  
collision  
(B)  $v_f = ?$

Later  
(C)



(1st) momentum

$$\vec{P}_A = \vec{P}_B \rightarrow +$$

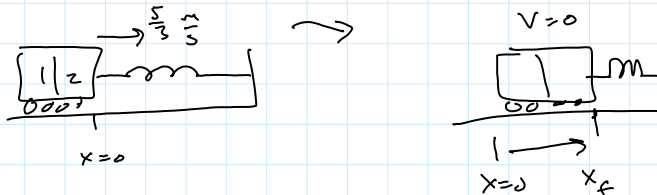
$$P_{1A} + P_{2A} = (P_{1+2})_B$$

$$m_1 v_{1A} + m_2 v_{2A} = m_{1+2} v_B$$

$$(10)(5) + 0 = (10 + 20) v_B$$

$$v_B = \frac{5}{3} \frac{\text{m}}{\text{s}}$$

Energy:



$$E_B = E_C$$

$$K_B + (U_{sp})_B = K_C + (U_{sp})_C$$

$$\frac{1}{2} m_{1+2} v_B^2 + 0 = 0 + \frac{1}{2} k x_f^2$$

↑
↑

Spring Not compressed
Not moving at time C

$$\frac{1}{2} (10+20) \left(\frac{5}{3}\right)^2 = \frac{1}{2} (50) x_f^2$$

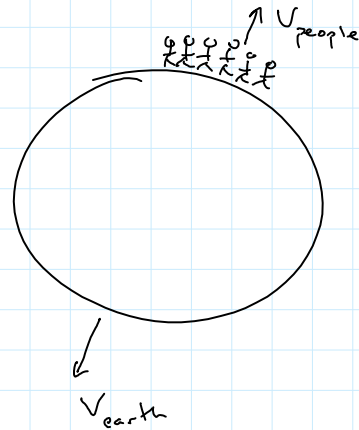
$$x_f = 1.29 \text{ m}$$

Every person on Earth meets in one place and jumps at the same time. What is the speed of the earth?

$$\vec{P}_{\text{people}} + \vec{P}_{\text{earth}} = 0$$

$$|\vec{P}_{\text{people}}| = |\vec{P}_{\text{earth}}|$$

$$M_{\text{people}} V_{\text{people}} = M_{\text{earth}} V_{\text{earth}}$$



$$M_{\text{earth}} = 6 \times 10^{24} \text{ kg}$$

$$M_{\text{people}} = (7 \times 10^9) (100 \text{ kg}) \approx 10^{12}$$

$V_{\text{people}}$ :

$$F_i = E_f$$

$$\frac{1}{2} m v^2 = mgh$$

$$v = \sqrt{2gh}$$

$$h = 1 \text{ m}$$

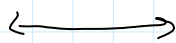
$$v = \sqrt{20} \approx 5 \frac{\text{m}}{\text{s}}$$

$$(10^{12}) (5) = (6 \times 10^{24}) V_{\text{earth}}$$

$$V_{\text{earth}} = 10^{-12} \frac{\text{m}}{\text{s}}$$

$$= 0.0000000000001 \frac{\text{m}}{\text{s}}$$

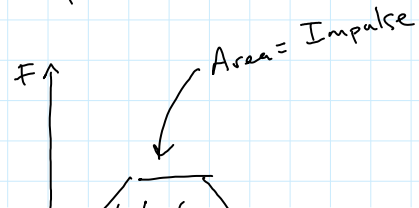
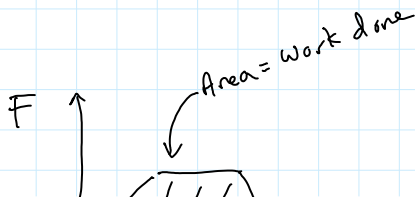
Energy



Momentum

Work

Impulse



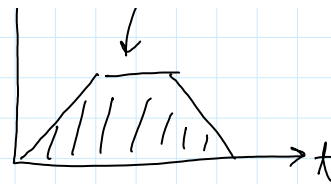


simple case:  $W = \vec{F} \cdot \Delta \vec{x}$   
(constant force)

Work-KE Theorem:

$$W_{\text{Net}} = \Delta KE$$

$$= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$



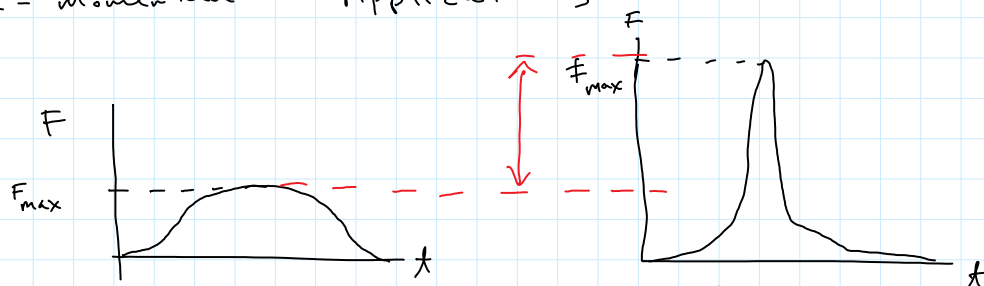
constant force:  $\vec{I} = \vec{F}_{\text{Ave}} \Delta t$

Impulse-Momentum Theorem:

$$\vec{I} = \Delta \vec{p}$$

$$= m \vec{v}_f - m \vec{v}_i$$

Impulse-momentum: Applications



Same area  
Same impulse  
Same  $\Delta \vec{p}$

gun recoil



$$\Delta p_A = \Delta p_B$$

$$F_{\text{Ave}} \Delta t = F_{\text{Ave}} \Delta t$$

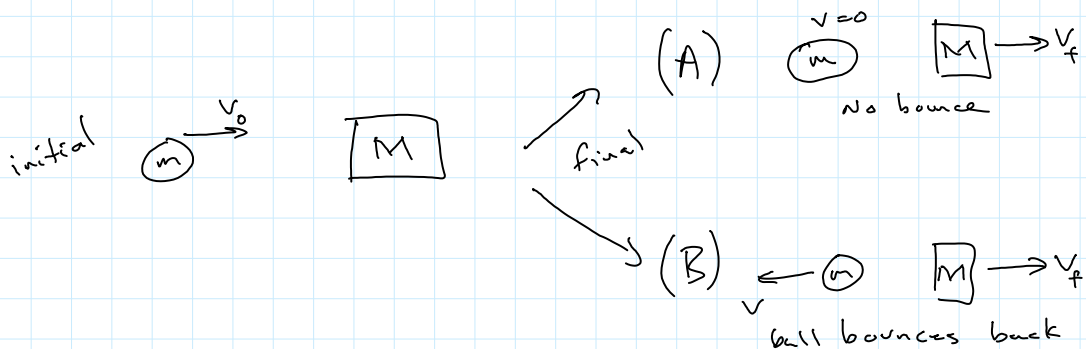
$$F_{\text{ave}} \Delta t = \int_{\text{ave}} dt$$

better for you

	<u>initial</u>	<u>final</u>	$\Delta \vec{p}$
a)	$2 \text{ kg}$ $0 \rightarrow 3 \frac{\text{m}}{\text{s}}$	$0 \rightarrow 5 \frac{\text{m}}{\text{s}}$	$p_f - p_i \rightarrow +$ $2(5) - (2)(3)$ $10 - 6$ $4 \text{ kg} \frac{\text{m}}{\text{s}}$ to the right

b)	$2 \text{ kg}$ $0 \rightarrow 3 \frac{\text{m}}{\text{s}}$	$5 \frac{\text{m}}{\text{s}} \leftarrow 0$	$p_f - p_i$ $(2)(-5) - (2)(3)$ $-10 - 6$ $-16 \text{ kg} \frac{\text{m}}{\text{s}}$ or $16 \text{ kg} \frac{\text{m}}{\text{s}}$ to the left
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c)	$2 \text{ kg}$ $0 \rightarrow 3 \frac{\text{m}}{\text{s}}$	$0$ $\downarrow 5 \frac{\text{m}}{\text{s}}$	$\Delta p_x = (p_x)_f - (p_x)_i \rightarrow +$ $= 0 - (2)(3)$ $= -6$ $\Delta p_y = (p_y)_f - (p_y)_i \downarrow +$ $= (2)(5) - 0$ $= 10$
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In which case does the block have a greater  $V_f$ ?

Case A

$$P_i = +5 \quad \text{all from ball}$$

$$P_f = +5 \quad \text{all from the block}$$

Case B

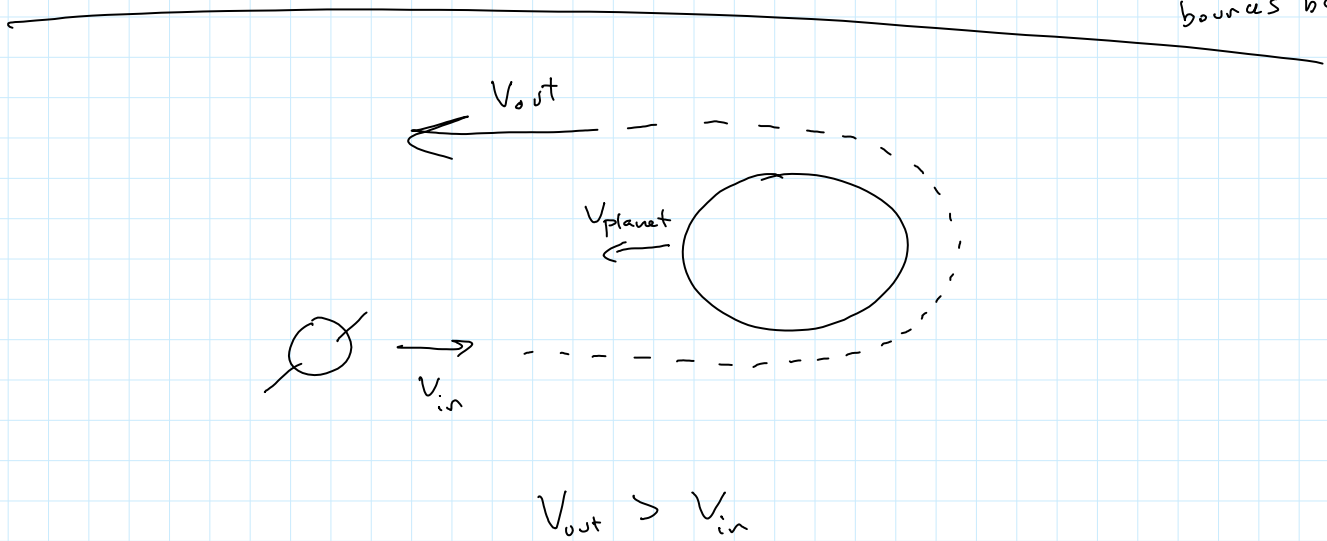
$$P_i = +5 \quad \text{all from ball}$$

$$(P_f)_{\text{ball}} = -3 \quad \text{ball bounces backwards}$$

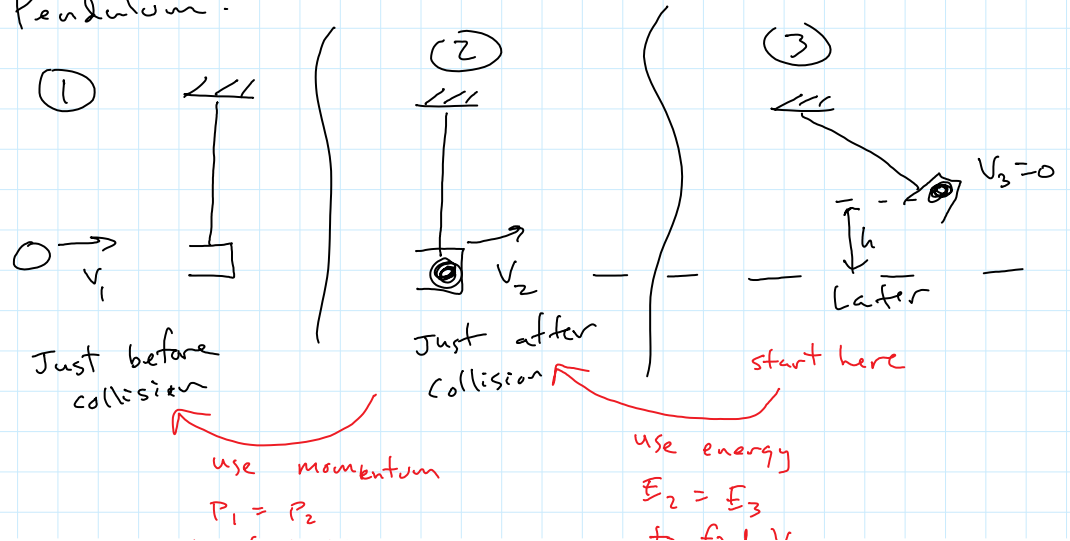
$$P_i = (P_f)_{\text{ball}} + (P_f)_{\text{block}}$$

$$(P_f)_{\text{block}} = +5 - (-3)$$

$$= +8 \quad \text{comes out faster if ball bounces back}$$



Ballistic Pendulum:



use momentum  
 $P_1 = P_2$   
to find  $V_1$

use energy  
 $E_2 = E_3$   
to find  $V_2$

Find Projectile speed ( $V_1$ )

given:  $m_{\text{ball}} = 0.05 \text{ kg}$

$m_{\text{pendulum}} = 0.15 \text{ kg}$

$h = 0.2 \text{ m}$

Working backwards

1<sup>st</sup>: Use Energy

$$E_2 = E_3$$

$$K_2 + (U_g)_2 = K_3 + (U_g)_3$$

$$\frac{1}{2}(m_b + m_p) V_2^2 = (m_b + m_p) g h$$

$$V_2 = \sqrt{2gh}$$

$$= \sqrt{2(9.8)(0.2)}$$

$$= 1.98 \frac{\text{m}}{\text{s}}$$

2<sup>nd</sup>: Use momentum

$$\vec{P}_1 = \vec{P}_2 \rightarrow +$$

$$(\vec{P}_b)_1 + (\vec{P}_p)_1 = (\vec{P}_{b+p})_2$$

$$m_b V_1 + 0 = (m_b + m_p) V_2$$

$$V_1 = \left( \frac{m_b + m_p}{m_b} \right) V_2$$

$$= \left( \frac{0.05 + 0.15}{0.05} \right) 1.98$$

$$= 7.92 \frac{\text{m}}{\text{s}}$$