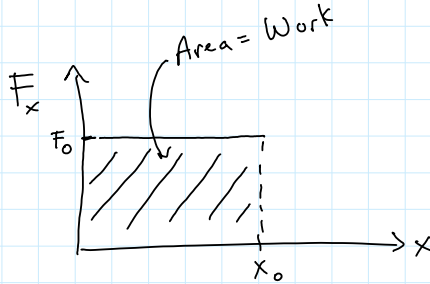


Goals for the Lecture:

- 1) Be able to calculate work done by constant forces
- 2) Be able to calculate the scalar product (dot product) of two vectors
- 3) Use the Work – Kinetic Energy Theorem to solve problems
- 4) Understand how defining your system can affect the work done on the system or by the system
- 5) Calculate kinetic and potential energy

Work

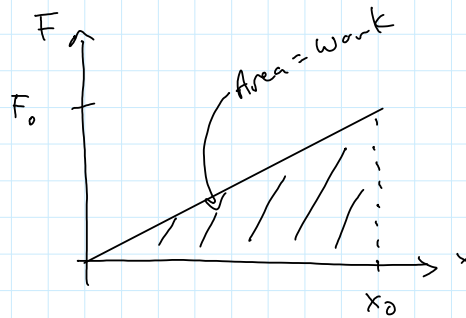
Constant force

work done on the object by F_0 :

$$W = \text{Area under } F \text{ vs } x \text{ curve}$$

$$= F_0 x_0$$

Non-constant force (like a spring)



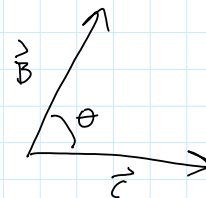
$$W = \text{Area under curve}$$

$$= \frac{1}{2} x_0 F_0$$

Dot Product:

$$A = \vec{B} \cdot \vec{C}$$

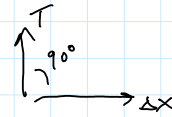
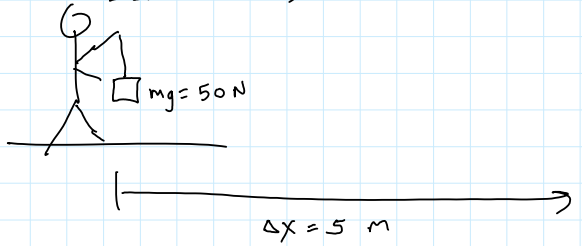
$$= |\vec{B}| |\vec{C}| \cos \theta$$



$$\text{Work: } W = \vec{F} \cdot \Delta \vec{x} \quad \text{For a constant force}$$

Find the work done on the box by the string: (box always moves at constant speed)

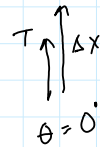
1) walking slowly at constant speed



$$W = T \Delta x \cos 90^\circ = 0$$

2)

box is lifted 0.8 m at constant speed

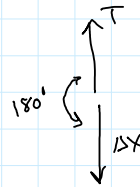
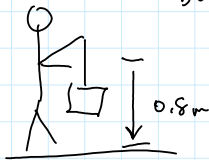


$$W = T \Delta x \cos 0^\circ$$

$$= T \Delta x = (50 \text{ N})(0.8) = 40 \text{ J}$$

3)

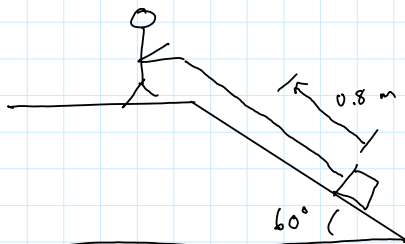
box is lowered 0.8 m at constant speed



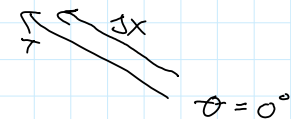
$$W = T \Delta x \cos 180^\circ$$

$$= -40 \text{ J}$$

4)



pulled up incline at constant speed

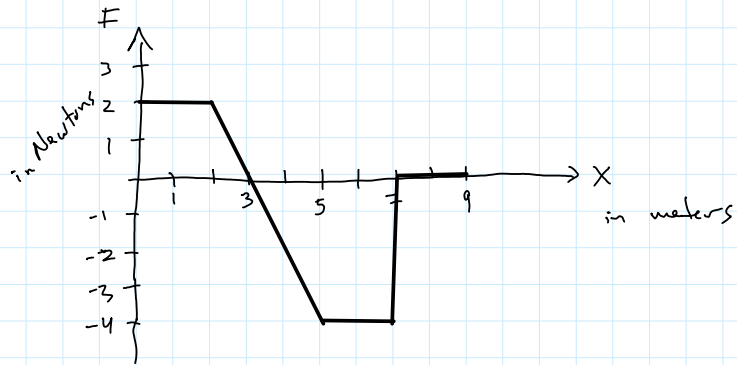


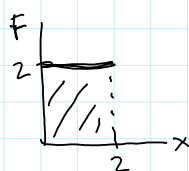
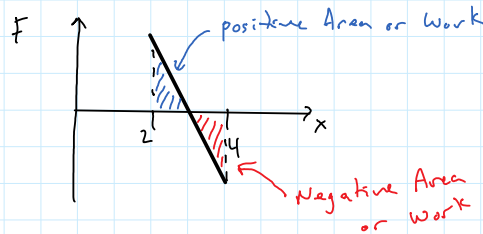
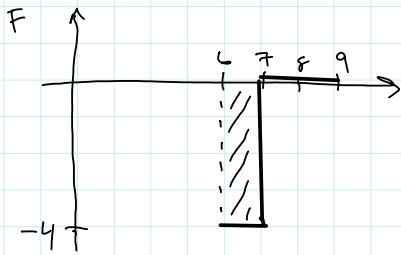
$$W = T \Delta x \cos 0^\circ$$

$$= 50 (0.8)$$

= 40 J

Find the work done on an object by F for each of the intervals:

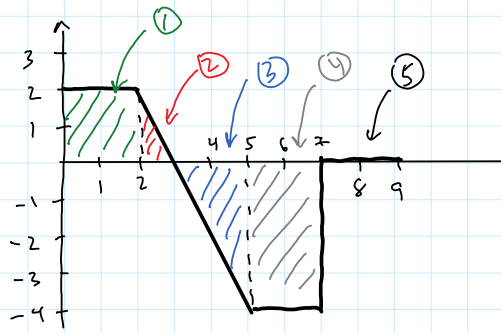


interval	work done on the object	Did it gain or lose speed?
$0 < x < 2$	 $W = \text{Area} = (2)(2) = 4 \text{ J}$	gained (W is positive)
$2 < x < 4$	 $W = +1 - 1 = 0$	stays the same speed
$6 < x < 9$	 $W = W_{6 \rightarrow 7} + W_{7 \rightarrow 9}$ $= -4 + 0$ $= -4 \text{ J}$	slowed down (negative work)

$0 < x < 9$

↑ ①

$$0 < x < 9$$



Area under the curve

$$\textcircled{1} \quad A_1 = (2)(2) = 4 \text{ J}$$

$$\textcircled{2} \quad A_2 = \frac{1}{2}(1)(2) = 1 \text{ J}$$

$$\textcircled{3} \quad A_3 = -\frac{1}{2}(2)(4) = -4 \text{ J}$$

$$\textcircled{4} \quad A_4 = -(2)(4) = -8 \text{ J}$$

$$\textcircled{5} \quad A_5 = 0$$

$$\text{Total Area} = W = (4 + 1 - 4 - 8 + 0) = -7 \text{ J}$$

Use work to find v_f , given: $m = 10 \text{ kg}$

$$g = 9.8 \text{ m/s}^2$$

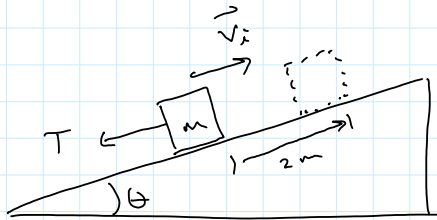
$$\theta = 30^\circ$$

$$v_i = 12 \frac{\text{m}}{\text{s}}$$

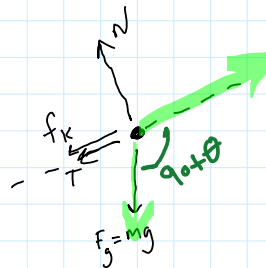
$$\Delta x = 2 \text{ m}$$

$$T = 5 \text{ N}$$

$$\mu_k = 0.15$$



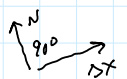

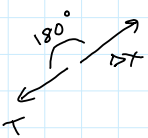
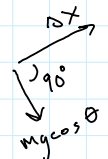
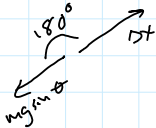
1) Draw FBD:



OR



2) Find work done on the box by every force acting on the box

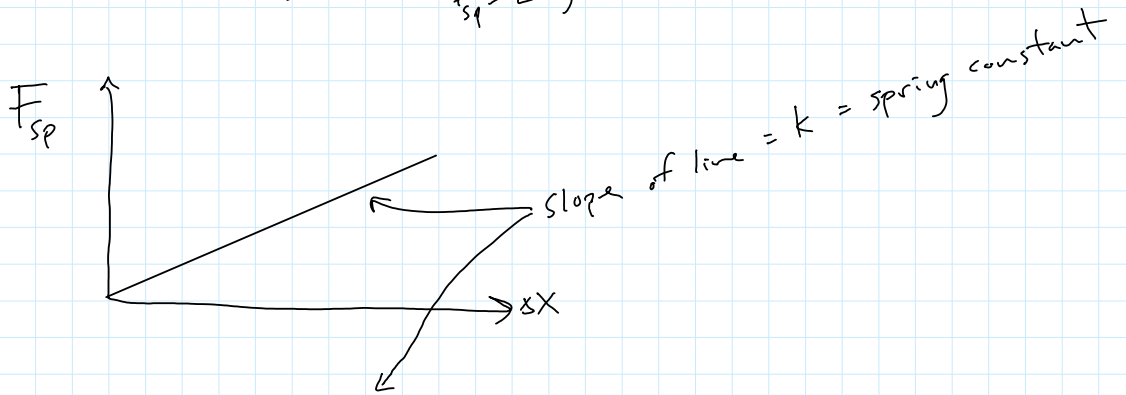
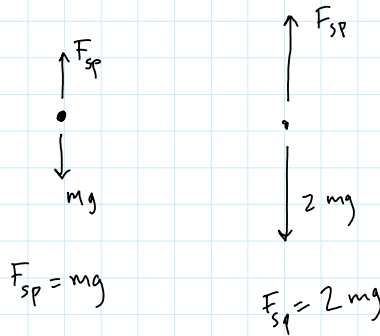
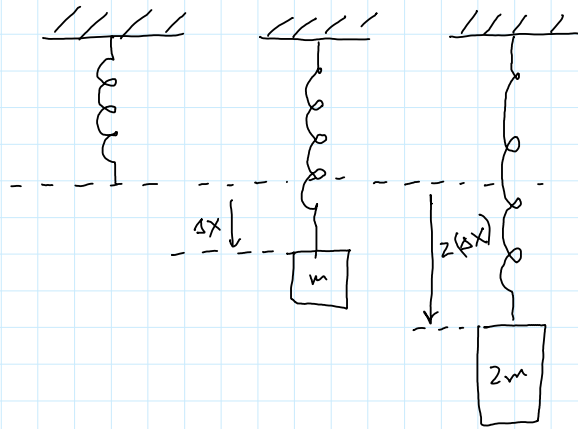
Force	Work done on box
N	 $W = N (\Delta x) \cos 90^\circ = 0$
f_k	 $ \begin{aligned} W &= f_k (\Delta x) \cos 180^\circ \\ &= \mu_k N \Delta x (-1) \\ &= (0.15) (mg \cos \theta) (2) (-1) \\ &= (0.15) (10) (9.8) (\cos 30) (2) (-1) \\ &= -25 \text{ J} \end{aligned} $ <p style="color: red; margin-left: 200px;">different angles</p>
T	 $ \begin{aligned} W &= T (\Delta x) \cos 180^\circ \\ &= (5) (2) (-1) \\ &= -10 \text{ J} \end{aligned} $
$(F_g)_\perp$	 $ \begin{aligned} W &= (mg \cos \theta) (\Delta x) \cos 90^\circ \\ &= 0 \end{aligned} $
$(F_g)_\parallel$	 $ \begin{aligned} W &= (mg \sin \theta) (\Delta x) \cos 180^\circ \\ &= (10) (9.8) (\sin 30) (2) (-1) \\ &= -98 \text{ J} \end{aligned} $

$$\begin{aligned}
 W_{\text{net}} &= W_N + W_{f_k} + W_T + W_{F_g} = 0 - 25 - 10 + 0 - 98 \\
 &= -133 \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 W_{\text{net}} &= \Delta K \\
 &= K_f - K_i \\
 &= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \\
 -133 &= \frac{1}{2} (10) v_f^2 - \frac{1}{2} (10) (12)^2
 \end{aligned}$$

$$V_f = 10.8 \frac{m}{s}$$

Springs:



Hooke's Law: $F_{sp} = k(\Delta X)$

vector form: $\vec{F}_{sp} = -k \vec{\Delta X}$

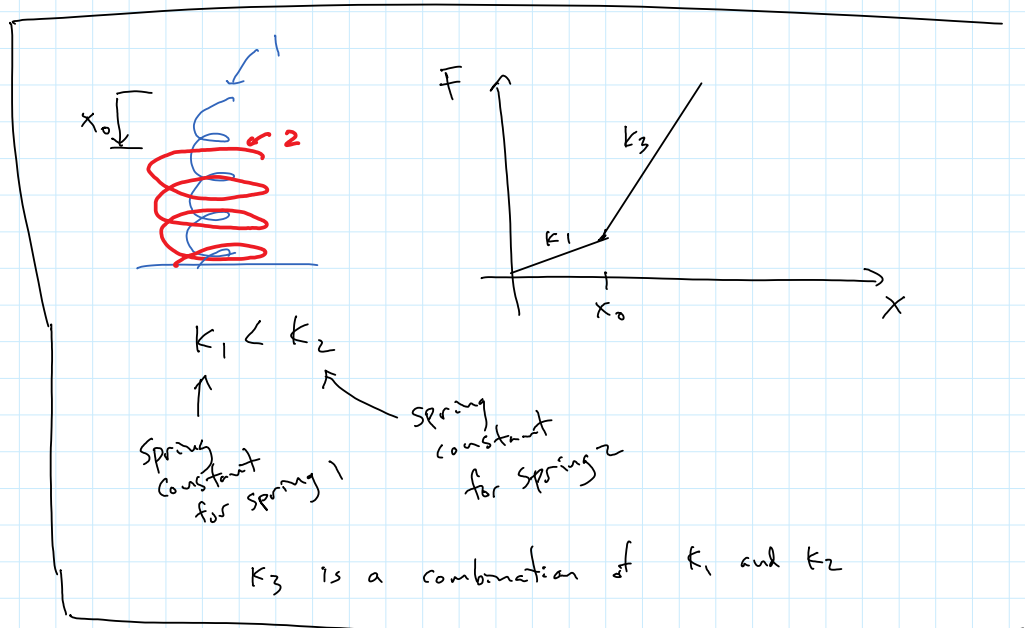
$$F_{sp} = k \Delta X$$

or

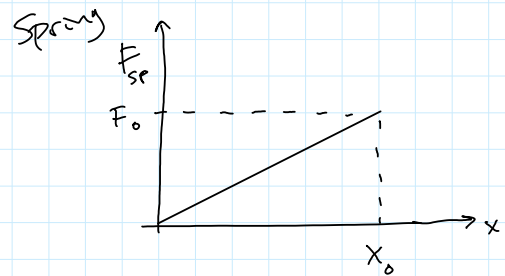
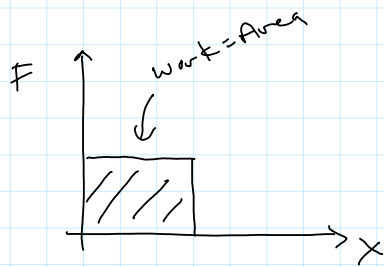
$$F_{sp} = k X$$

Amount spring is stretched or compressed

$F_{sp} = kx$
 ↑
 Spring constant
 (depends on the spring)



Work



$$W = \frac{1}{2} x_0 F_0$$

↑
 $F_0 = kx_0$

$$W = \frac{1}{2} k x_0^2$$

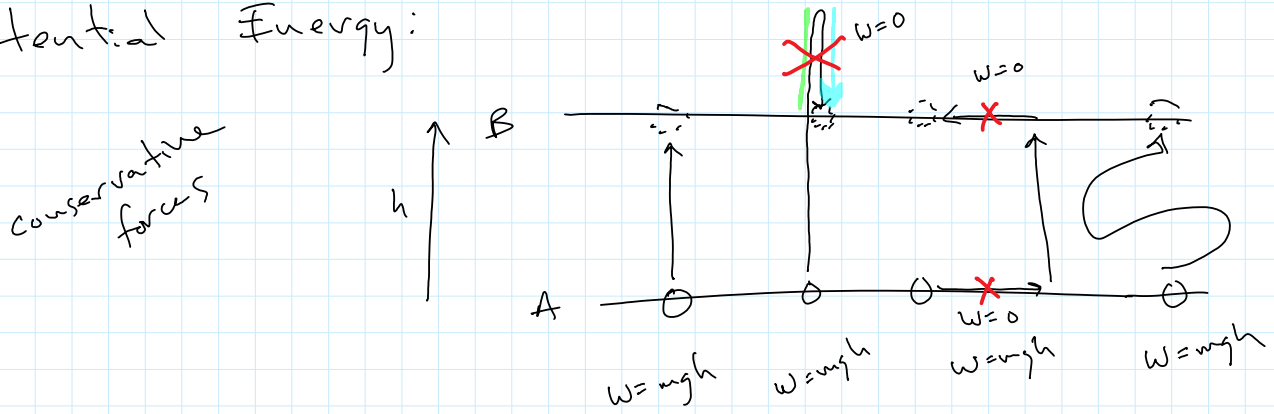
In general:

$$W_{\text{Spring}} = \pm \frac{1}{2} k x^2$$

Spring Examples

initial	final	Work done on the box by the spring
<p>1)</p>		$ W = \frac{1}{2} (10)(2)^2$ $ W = 20 \text{ J}$ W is Negative in this case (slowing the box down) $W = -20 \text{ J}$
<p>2)</p>		$W = \pm \frac{1}{2} k x^2$ in general $W = + \frac{1}{2} k x^2$ in this case W is positive (speeding the box up) $W = \frac{1}{2} (10) 2^2$ $= +20 \text{ J}$

Potential Energy:



For every conservative force we make a

For every conservative force we make a potential energy term:

gravity: $U_g = mgy$

can be positive or negative

Spring: $U_{sp} = \frac{1}{2} kx^2$

Always positive

Using Energy to solve Problems:

$$E = \text{mechanical energy}$$
$$= K + U_g + U_{sp}$$

$$E_i + W_{\text{friction}} + W_{\text{ext}} = E_f$$

↑
since this is Negative it goes on left side of equation