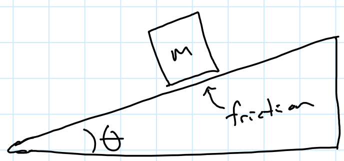
Newton's Laws Problems & Solutions

Friday, February 3, 2017 10:58 AM

Incline:



given: $m = 10$ kg

$\theta = 30^\circ$

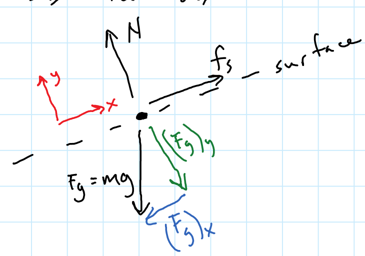
$\mu_s = 0.4$

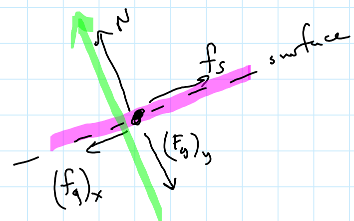
$\mu_k = 0.2$

Question:

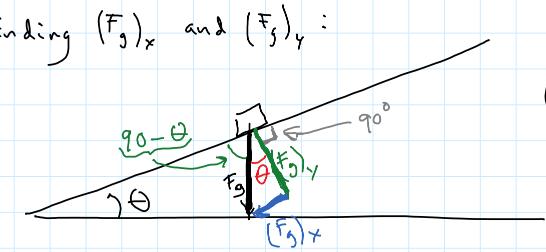
Tell me if the box begins to slide down incline if it starts at rest

1st) FBD for box





Finding $(F_g)_x$ and $(F_g)_y$:



$$(F_g)_y = F_g \cos\theta$$

$$(F_g)_x = F_g \sin\theta$$

2nd) $\Sigma \vec{F} = m\vec{a}$

$\Sigma F_x = m a_x$ +

$\underbrace{f_s}$ $- (F_g)_x = m a_x$

is this Negative when f_s hits its maximum value

$\Sigma F_y = m a_y$ +

$N - (F_g)_y = m \cancel{a_y}^{0}$

$N = (F_g)_y$

$N = mg\cos\theta$

OR

if $(F_g)_x > (f_s)_{max}$ it slides down

if $(F_g)_x \leq (f_s)_{max}$ it stays put

$$(F_g)_x = mg\sin\theta = (10 \text{ kg})(10 \tfrac{m}{s^2}) \sin 30^\circ = 50 \text{ N}$$

$$(f_s)_{max} = \mu_s N = \mu_s mg\cos\theta = (0.4)(10)(10)\cos 30^\circ = 34\text{ N}$$

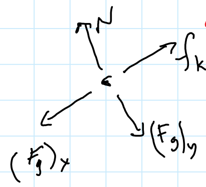
$50 > 34$ it slides down !

Follow-up question:

Find the acceleration of this block as it slides down the incline.

$\Sigma F_x = m a_x$ $\swarrow +$

I know $\vec{a}$ points down incline and I like to make the positive direction the same direction as the direction of the acceleration



this is now kinetic friction because the box is sliding

$$(F_g)_x - f_k = m a$$

$$mg\sin\theta - \mu_k N = ma$$

$$mg\sin\theta - \mu_k(mg\cos\theta) = ma$$

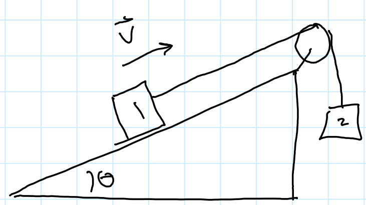
$$a = g\sin\theta - \mu_k g\cos\theta$$

$$= 9.8 \sin 30^\circ - (0.2)(9.8)\cos 30^\circ$$

[Now I'm using $g = 9.8\ \frac{m}{s^2}$]

$$= 3.20\ \frac{m}{s^2} \text{ down incline}$$

Pulley



given: $m_1 = 10$ kg

$m_2 = 20$ kg

$\theta = 30^\circ$

$\mu_k = 0.2$

massless pulley

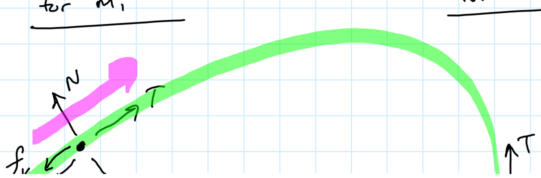
Find: T in rope

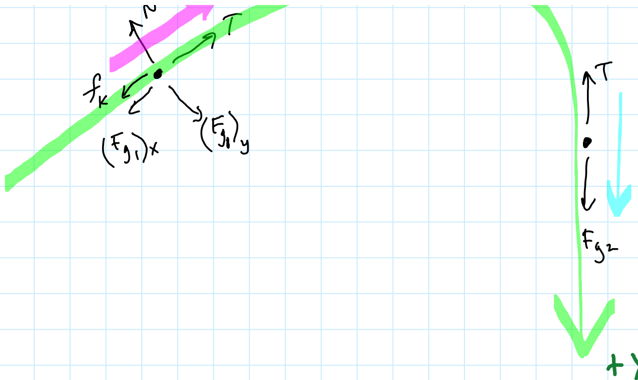
and

a acceleration

FBD for m_1

FBD for m_2





m_1

$$\Sigma F_x = m_1 a_x \quad \rightarrow +$$

$$T - f_k - (F_{g1})_x = m_1 a$$

$$T - \mu_k N - m_1 g \sin\theta = m_1 a$$

m_1

$$\Sigma F_y = m_1 \cancel{a_y}^{\,0}$$

$$N - (F_{g1})_y = 0$$

$$N = m_1 g \cos\theta$$

m_2

$$\Sigma F = m_2 a_x \quad \downarrow +$$

$$F_{g2} - T = m_2 a$$

$$m_2 g - T = m_2 a$$

$$T = m_2 g - m_2 a$$

$$T - \mu_k m_1 g \cos\theta - m_1 g \sin\theta = m_1 a$$

$$(m_2 g - m_2 a) - \mu_k m_1 g \cos\theta - m_1 g \sin\theta = m_1 a$$

$$(20)(9.8) - (20)a - (0.2)(10)(9.8)\cos 30^\circ - (10)(9.8)\sin 30^\circ = 10a$$

$$164 = 30a$$

$$a = 5.47 \ \frac{m}{s^2} \quad \text{up the incline for } m_1$$

To find T, use either of the above two equations.
I'll use the equation from box 2:

$$T = m_2 g - m_2 a$$

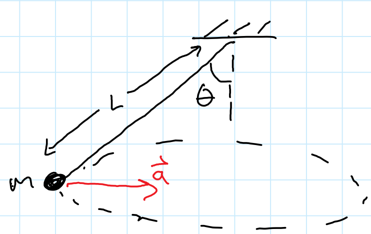
$$= (20)(9.8) - (20)(5.47)$$

$$= 86.7 \ N$$

Circular motion

m swings in a horizontal circle

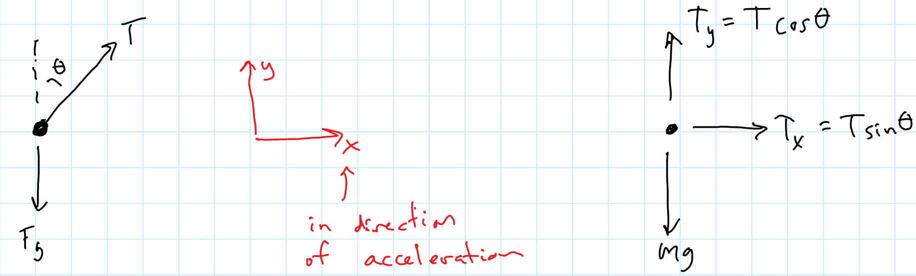
String makes a cone shape as it rotates



given: $\begin{cases} m = 0.3 \text{ kg} \\ \theta = 50^\circ \\ L = 0.7 \text{ m} \end{cases}$

Find : v speed of the mass
and
T tension in the string

FBD



$$\Sigma \vec{F} = m\vec{a}$$

$$\Sigma F_x = m a_x \quad \rightarrow +$$

toward center of circle

$$\Sigma F_y = m a_y \quad \uparrow +$$

$$T_y - F_g = 0$$

$$T\cos\theta = mg$$

$$T = \frac{mg}{\cos\theta}$$

$$T_x = m\, a_{cp}$$

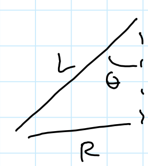
$$T\sin\theta = m\left(\frac{v^2}{R}\right)$$

$$\left(\frac{mg}{\cos\theta}\right)\sin\theta = m\frac{v^2}{R}$$

$$g\tan\theta = \frac{v^2}{R}$$

$$v = \sqrt{Rg\tan\theta}$$

Find R: radius of the circular path



$$\sin\theta = \frac{R}{L}$$

$$R = L\sin\theta$$

$$v = \sqrt{L\sin\theta\; g\tan\theta}$$

$$= \sqrt{(0.7)\sin 50\,(9.8)\tan 50}$$

$$= 2.50\ \frac{m}{s}$$

$$T = \frac{mg}{\quad} = \frac{(0.3)(9.8)}{\quad} = 4.57\ N$$

$$T = \frac{mg}{\cos\theta} = \frac{(0.3)(9.8)}{\cos 50^\circ} = 4.57 \text{ N}$$