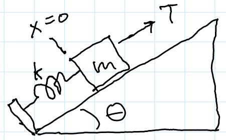
Energy Problems with Solutions

Tuesday, February 21, 2017 5:00 PM

1) Pulling a box up an incline



given: $m = 5$ kg

$T = 100$ N

$k = 50$ N/m

Spring starts at its equilibrium length

$\mu_k = 0.2$

$\theta = 30°$

$v_i = 0$

find: v_f after the box has moved up the incline a distance $D = 1.5$ m

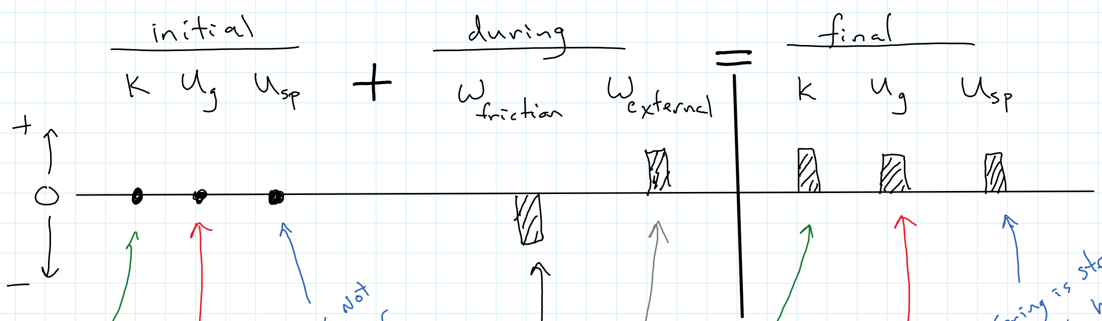
Define your system: in our system: m, spring, earth

external forces: T

This is important so you do Not double count F_g and F_{sp} effects. If something is in the system, it cannot do external work, but you need to use potential energy.
If it is Not included in the system, you should not have any potential energy terms for that force, but it will reach into the system and do work as an external force.

Typically, you want the earth and springs in your system so you can use potential energy.

Energy bar chart:



m is not moving, so kinetic energy is zero

Spring is Not stretched or compressed, so $U_{sp} = 0$ (No energy stored)

friction always Negative work

m is moving at end, so it has KE. Kinetic energy is always positive

Spring is stretched so, it has energy stored (Ug always positive, doesn't matter if spring is stretched or compressed)

m moves up, so we need to account for changes in Ug. Let's make the lowest height, so y=0 here

We have an external force. In this case it does positive work on our system

m ends up, up higher at end. so it must have more Ug at end. it started at must be here

to chang.
Let's make point y=0 heig
$U_g = 0$ here

extern
In this
does positive
on our system

so it U_g start be
more since it must
zero, it positive here

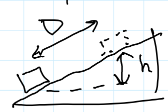
co

$$W_{friction} + W_{ext} = K_f + (U_g)_f + (U_{sp})_f$$

$$-f_k D + TD = \tfrac{1}{2} m v_f^2 + mgh + \tfrac{1}{2} k D^2$$

f_k = $\mu_k N$

spring stretched by D



$h = D \sin\theta$

$$-\mu_k \underbrace{(mg\cos\theta)}^{N} D + TD = \tfrac{1}{2} m v_f^2 + mg D \sin\theta + \tfrac{1}{2} k D^2$$

$$-(0.2)(5)(9.8)\cos 30^\circ (1.5) + (100)(1.5) = \tfrac{1}{2}(5) v_f^2 + (5)(9.8)(1.5)\sin 30^\circ + \tfrac{1}{2}(50)(1.5)^2$$

$$-12.7 + 150 = 2.5 v_f^2 + 36.8 + 56.3$$

$$v_f = 4.21 \ \tfrac{m}{s} \text{ up the incline}$$

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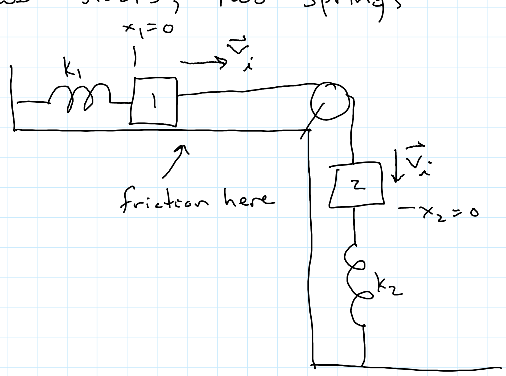
2) Two blocks, two springs



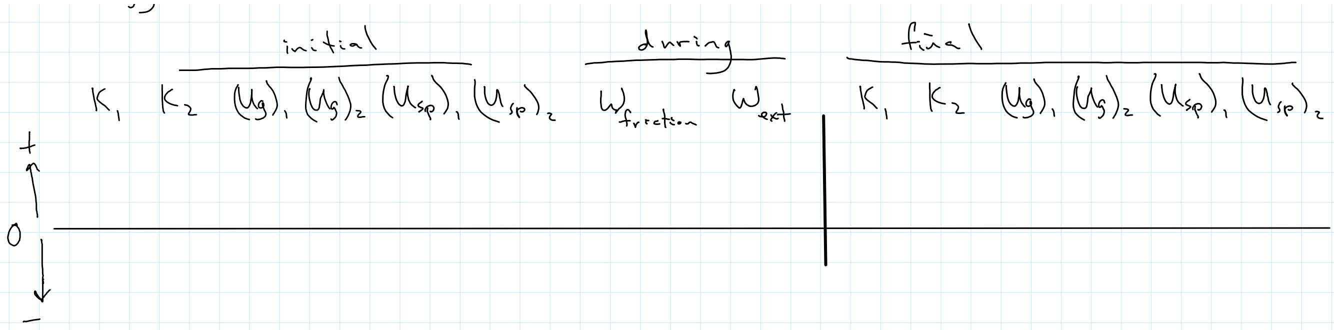
given:
$m_1 = 10$ kg
$m_2 = 20$ kg
$k_1 = 100 \ \frac{N}{m}$
$k_2 = 200 \ \frac{N}{m}$
$\mu_k = 0.2$
both springs start at their equilibrium positions
$v_i = 4 \ \frac{m}{s}$ as shown

find: distance, x, that spring 2 is compressed when the boxes come to rest

Define system: In system: m_1, m_2, spring 1, spring 2, earth
external forces: None

Energy bar chart:

initial	during	final

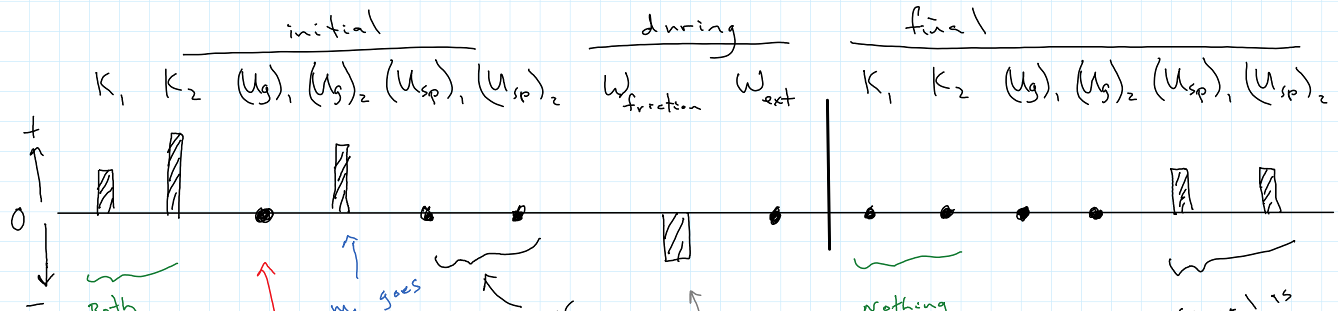


Note:

you need K_i and K_f for every mass in your system

you need $(U_g)_i$ and $(U_g)_f$ for every mass in your system

you Need $(U_{sp})_i$ and $(U_{sp})_f$ for every spring in your system



Both masses are moving initially. K is always +

m_1 doesn't go up or down. Let's just call its height $y = 0$, so $(U_g)_1 = 0$

m_2 goes down. Let's call final height zero so, $(U_g)_i$ is positive

Neither spring is stretched or compressed

friction always does Negative work on system

Nothing moving at end

Spring 1 is stretched and spring 2 is compressed. Both store energy. U_{sp} is always positive

$$\frac{1}{2}m_1v_i^2 + \frac{1}{2}m_2v_i^2 + m_2 g x - f_x x = \frac{1}{2}k_1x^2 + \frac{1}{2}k_2x^2$$

distance m_2 goes down

distance m_1 moves along surface

distance spring 1 is stretched

distance spring 2 is compressed

$$\frac{1}{2}(10)(4)^2 + \frac{1}{2}(20)(4)^2 + (20)(9.8)x - \underbrace{(0.2)}_{\mu_k}\underbrace{(10)(9.8)}_{N}x = \frac{1}{2}(100)x^2 + \frac{1}{2}(200)x^2$$

$$150x^2 - 176.4x - 240 = 0$$

$$X = \begin{cases} -0.807 \text{ m} \\ 1.98 \text{ m} \quad \leftarrow \text{positive root} \end{cases}$$

$$X = 1.98 \text{ m}$$