The Remainder Theorem if $a$ divides $b$ with quotient $c$ and remainder $d$, then $b = a \times c + d$

**example:** 3 divides 11 with quotient 3 and remainder 2
therefore $11 = 3 \times 3 + 2$

The polynomial version of the remainder theorem is called the Division Algorithm for Polynomials.

**Division Algorithm for Polynomials**
If $(x - a)$ divides $P(x)$ with quotient $q(x)$ and remainder $r(x)$, then $P(x) = (x - a) q(x) + r(x)$
where $\deg[P(x)] = n$
$\deg[q(x)] \leq n - 1$
$\deg[r(x)] \leq 1$

**example:** $(x - 1)$ divides $x^2 - 4x + 9$ with quotient and remainder 6
therefore $x^2 - 4x + 9 = (x - 1)(x - 3) + 6$

**Factor Theorem**
The Factor Theorem states that if $(x - a)$ divides $P(x)$ without remainders (i.e. $r(x) = 0$), then $(x - a)$ must be a factor of $P(x)$, which can be represented as:
$P(x) = (x - a) q(x)$
where $\deg[P(x)] = n$
$\deg[q(x)] \leq n - 1$

**example:** $(x + 1)$ divides $x^3 + 2x^2 + 2x + 1$ without remainders
therefore $(x + 1)$ can be factored out from $x^3 + 2x^2 + 2x + 1$,
using synthetic division, we have
$x^3 + 2x^2 + 2x + 1 = (x + 1)(x^2 + x + 1)$

**Applications of the theorems**
**Check if $(x - a)$ a factor of $P(x)$**
To check if $(x - a)$ a factor of $P(x)$, that to check if $(x - a)$ divides $P(x)$
with a remainder = 0, using the Division Algorithm, substituting $x = a$,
$P(a) = (a - a) q(a) + r(a)$
the first term on the right hand side will become zero and therefore the remainder when $(x - a)$ divides $P(x)$, $r(a)$ is equal to $P(a)$. 
example: Is \((x - 2)\) a factor of \(P(x) = x^3 + 2x^2 + 2x + 1\)?
To check this, we put \(x = 2\) and compute the value of the polynomial.

\[
P(2) = (2)^3 + 2(2)^2 + 2(2) + 1 = 21
\]
Since \(P(2)\) does not equal to 0, \((x - 2)\) is not a factor of \(P(x)\).

To compute the value of \(P(k)\) (\(k\) is a constant)
This is the reverse process of the first application. Using the Division Algorithm, substituting \(x = k\),

\[
P(k) = (k - k)q(k) + r(k)
\]
the first term on the right hand side will become zero and therefore the functional value \(P(k)\) is equal to the remainder when \(P(x)\) is divided by \((x - k)\).

example: Find the value of \(P(k)\) for given conditions:
\(P(x) = f(x) + g(x)\)
The remainder of \(f(x)\) divided by \((x - k)\) is \(a\)
The remainder of \(g(x)\) divided by \((x - k)\) is \(b\)

Applying Division Algorithm on \(f(x)\) and \(g(x)\),

\[
f(x) = (x - k)q_1(x) + r_1(x)
g(x) = (x - k)q_2(x) + r_2(x)
\]
Substituting \(x = k\) on both sides,

\[
f(k) = (k - k)q_1(k) + r_1(k) = r_1(k)
g(k) = (k - k)q_2(k) + r_2(k) = r_2(k)
\]
Since \(r_1(k) = a\), \(r_2(k) = b\),

\[
f(k) = a
g(k) = b
\]
Therefore using the fact that \(P(x) = f(x) + g(x)\)

\[
P(k) = f(k) + g(k) = a + b
\]